Flows on Surfaces of Arbitrary Topology

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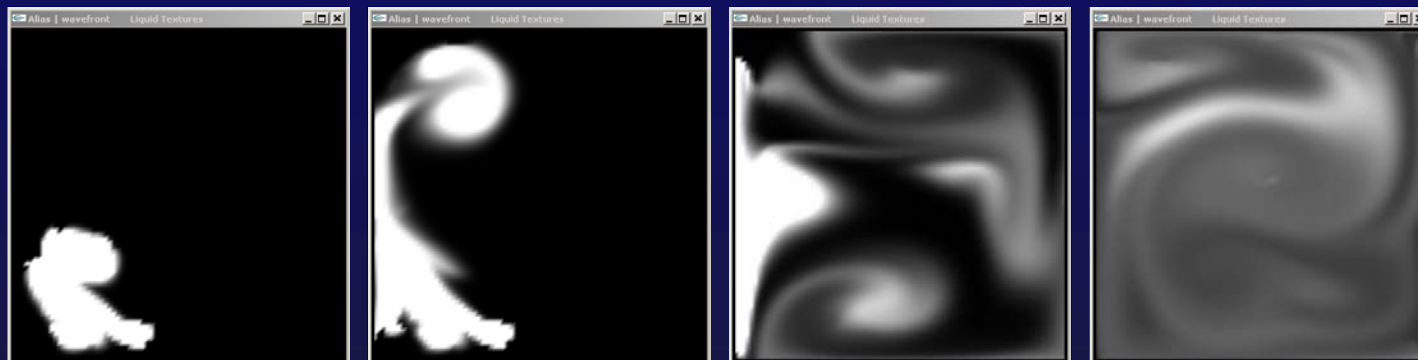
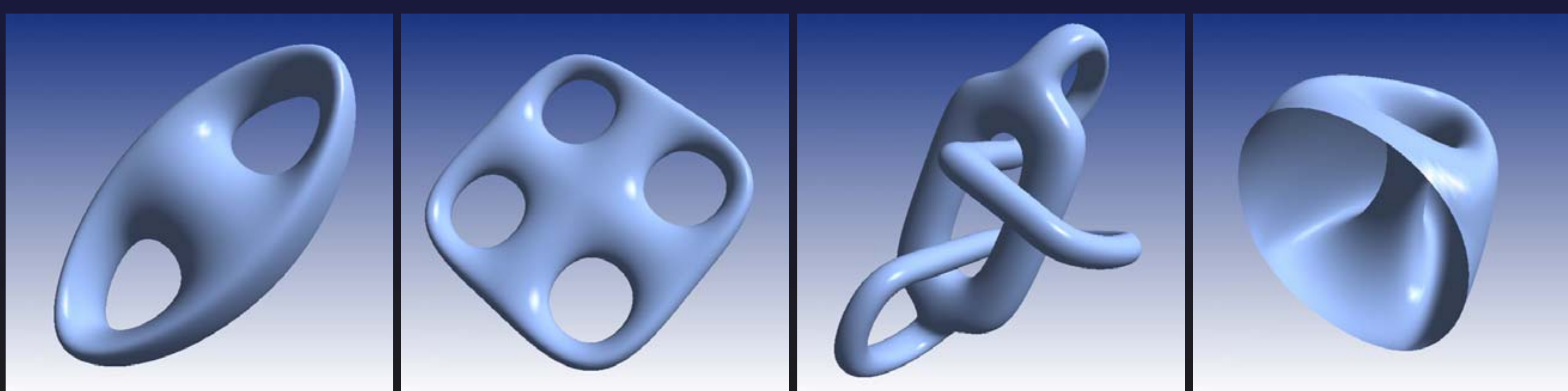
Alias

Basic Idea

Combine two pieces of code





Flow Model

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

Incompressible Navier-Stokes

Stable Fluids (SIGGRAPH'99)

Demo

Catmull-Clark Surfaces

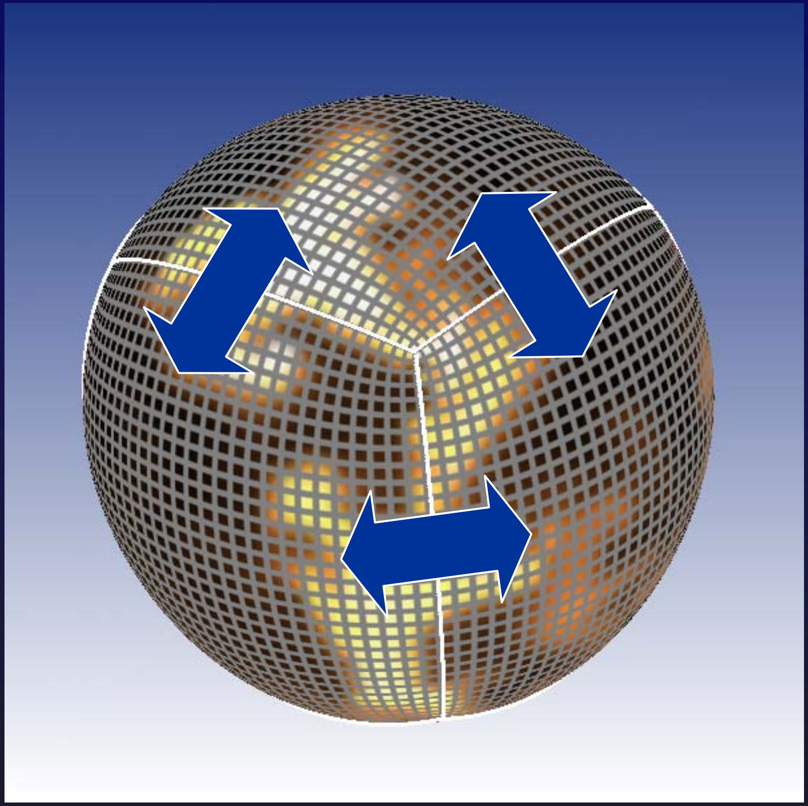
Catmull and Clark 1978.

Properties:

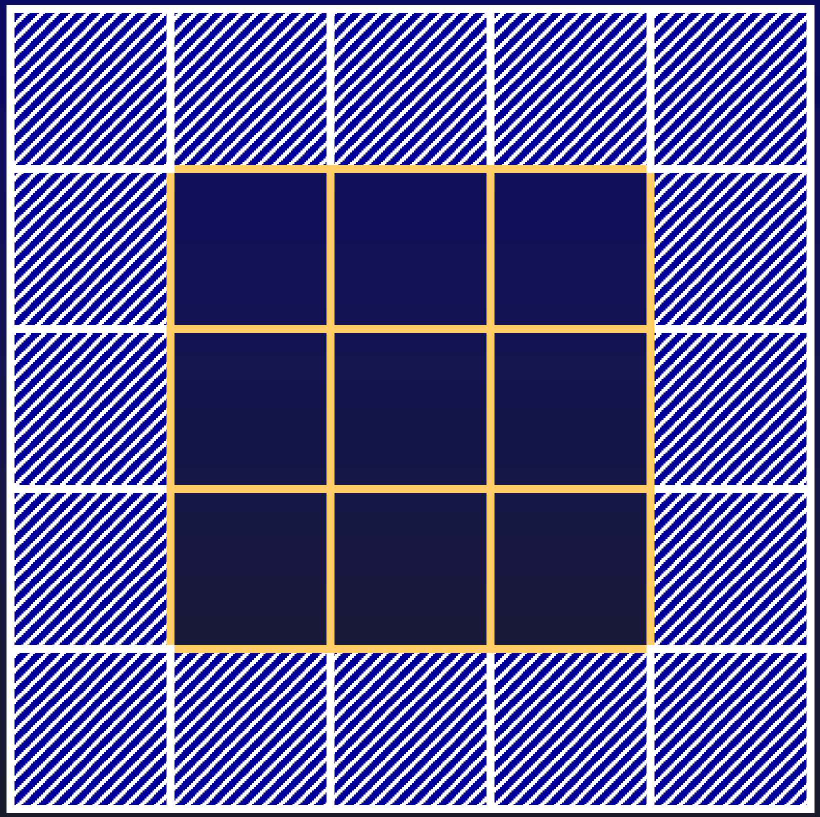
- Arbitrary Topology
- Smooth (C1) [Reif,Zorin,Peters]
- Exact Evaluation [siggraph'98]

Demo

Cross-Patch Interactions

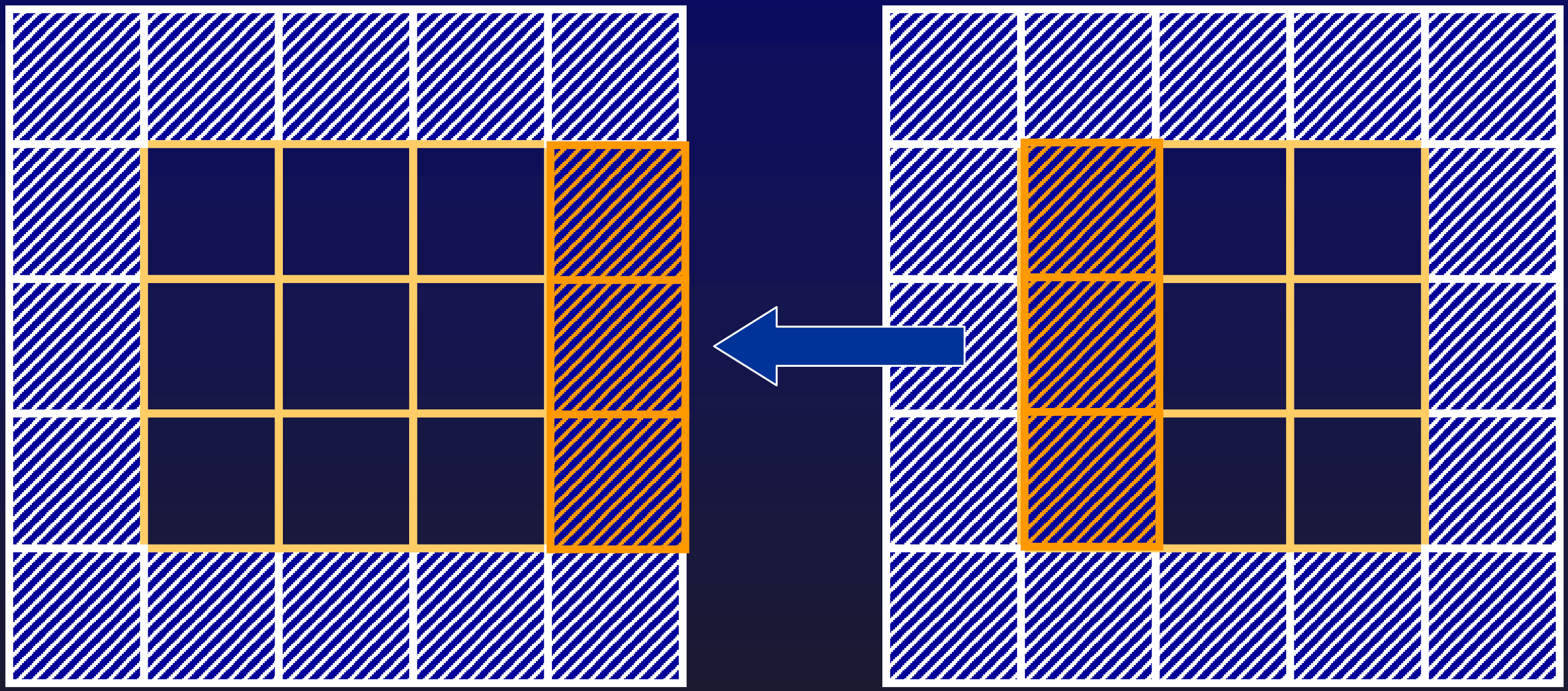


Cross-Patch Interactions



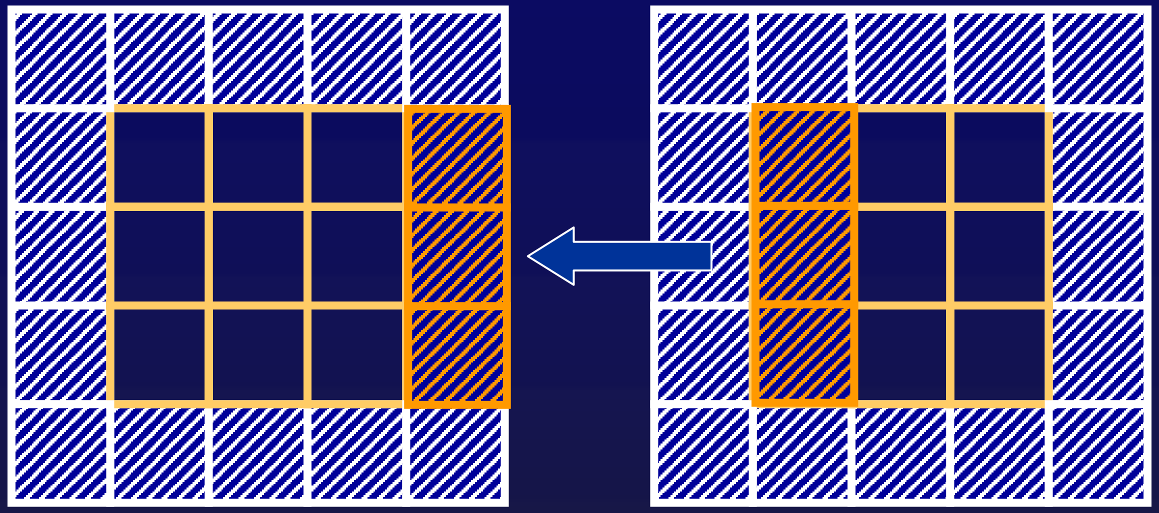
Add another layer of cells around each grid

Cross-Patch Interactions



Fill in boudary cells from neighbor patch after each update

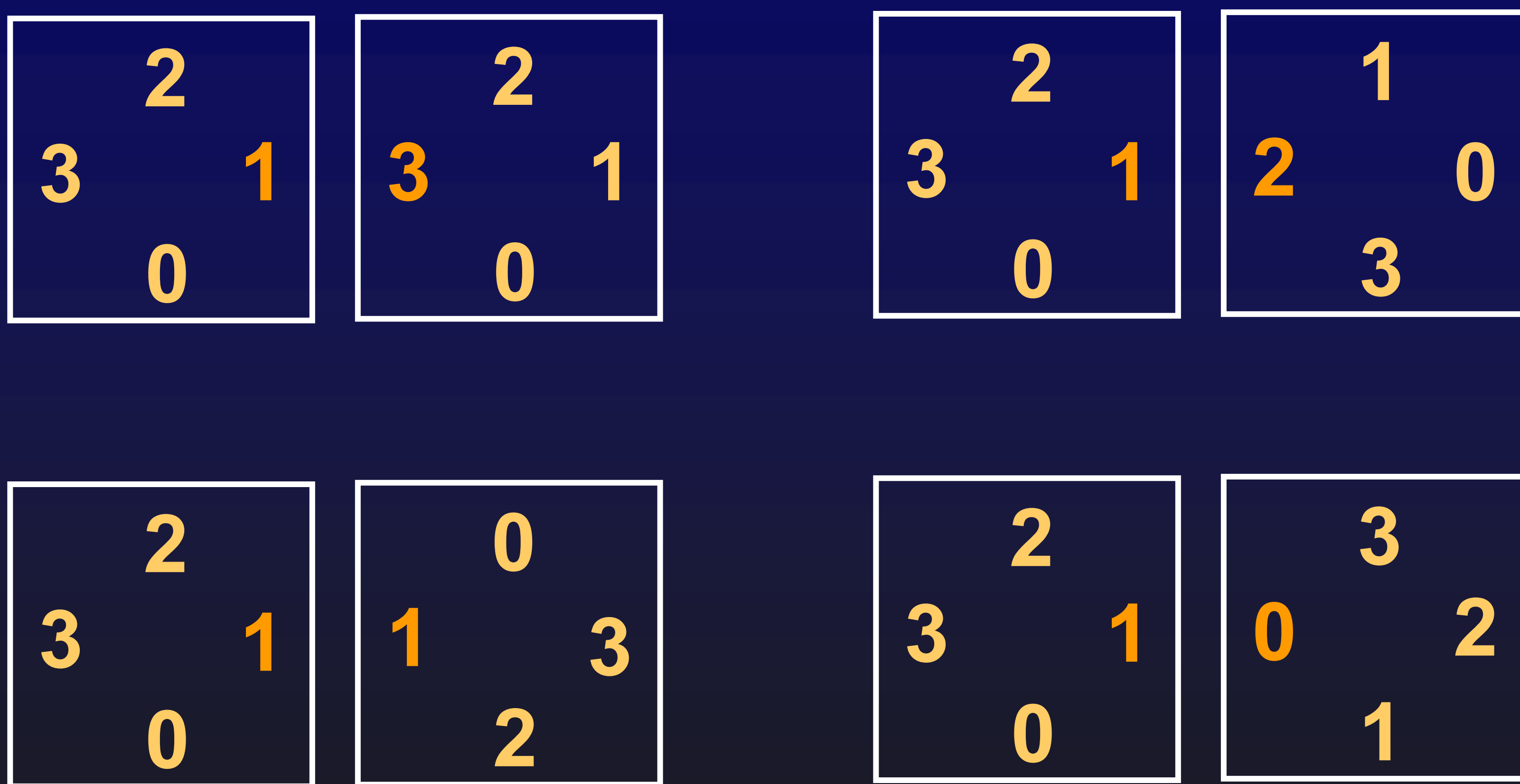
Cross-Patch Interactions



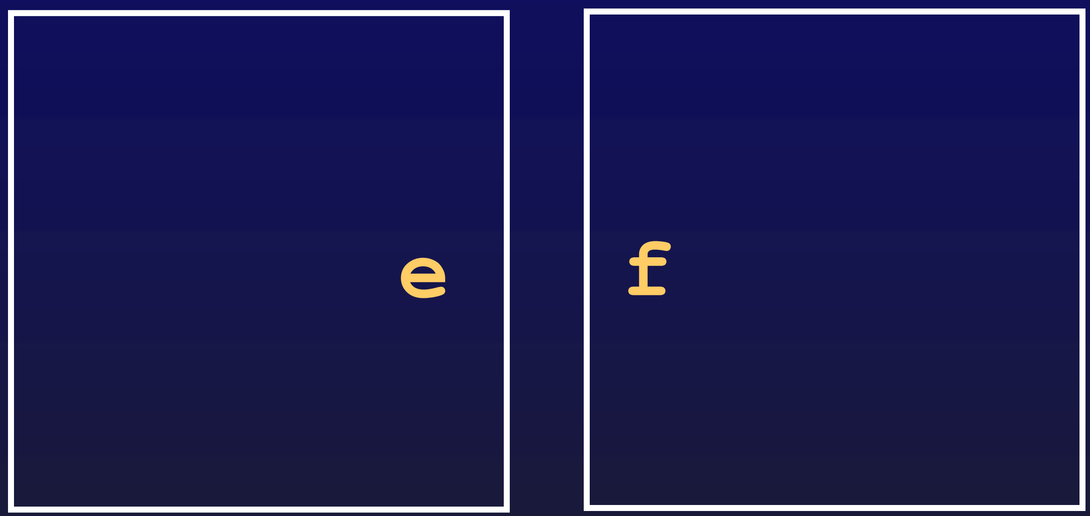
```
for ( i=1 ; i<=N ; i++ ) {
      A[N+1,i] = An[1,i];
}
```

Easy if Grids aligned

4 Cases / Edge = 16 Cases



4 Cases Enough !

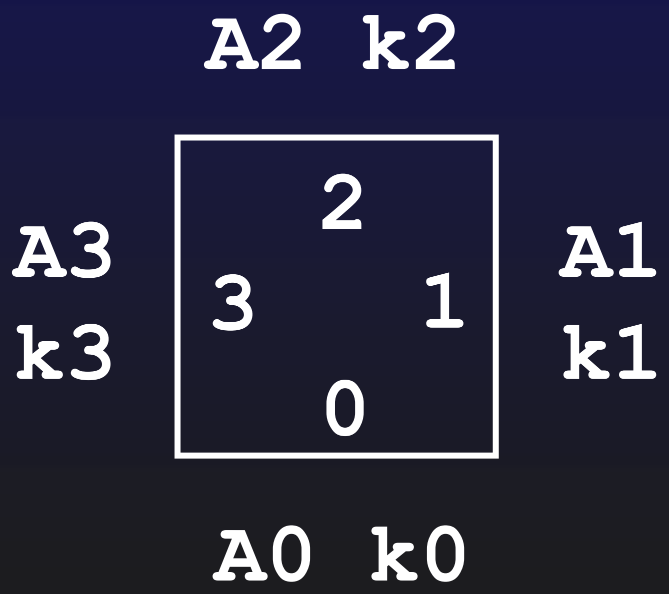


Transition code only depends on

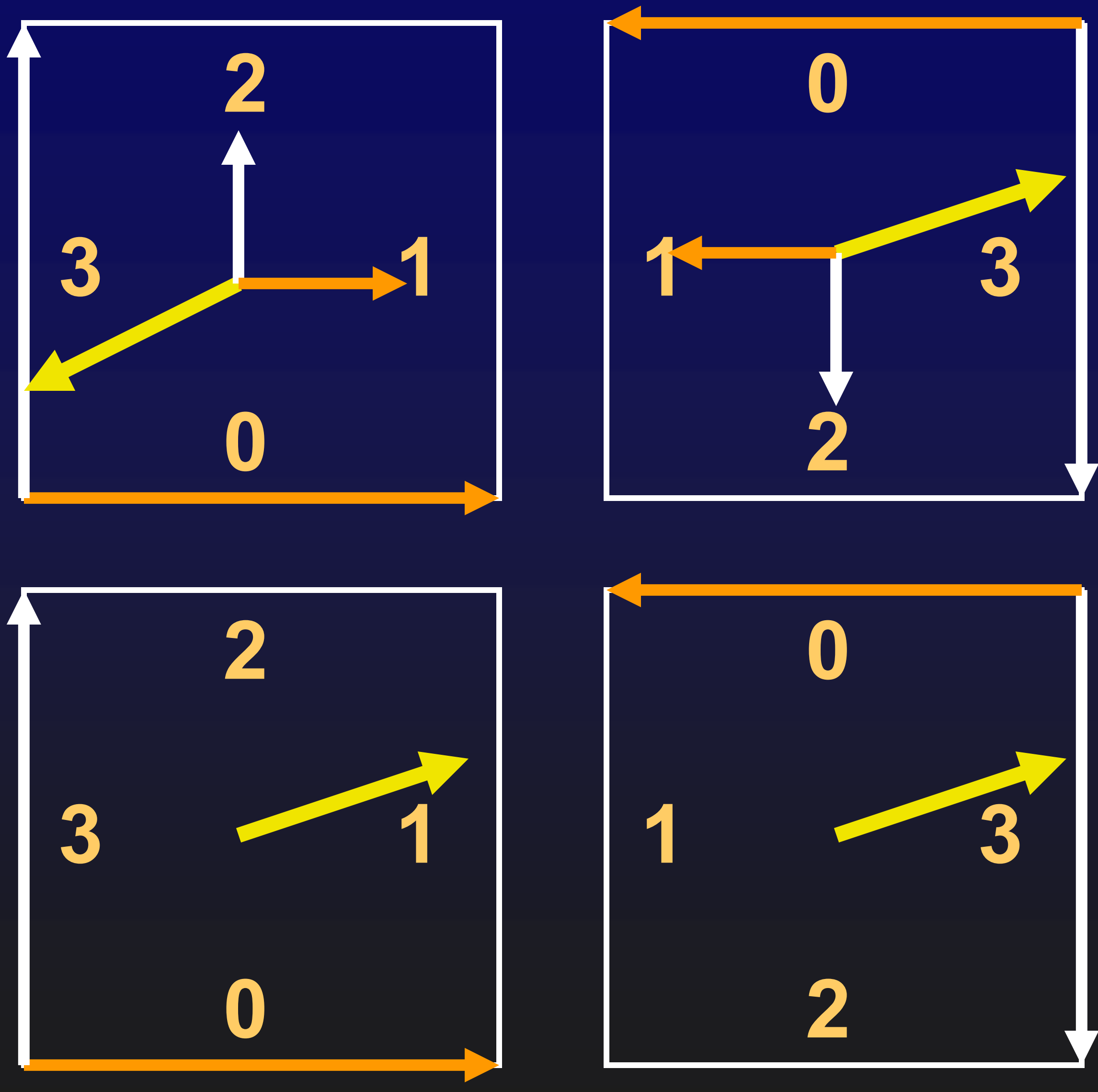
```
k = (4+e-(f+2)%4)%4
```

Simple code

```
Get neighbor arrays: A0, A1, A2, A3

Compute: k0, k1, k2, k3

for ( i=1 ; i<=N ; i++ ) {

   A[i,0]   = A0[idx(k0,i,N)];

   A[N+1,i] = A1[idx(k1,1,i)];

   A[i,N+1] = A2[idx(k2,i,1)];

   A[0,i]   = A3[idx(k3,N,i)];

}
```



VELOCITY



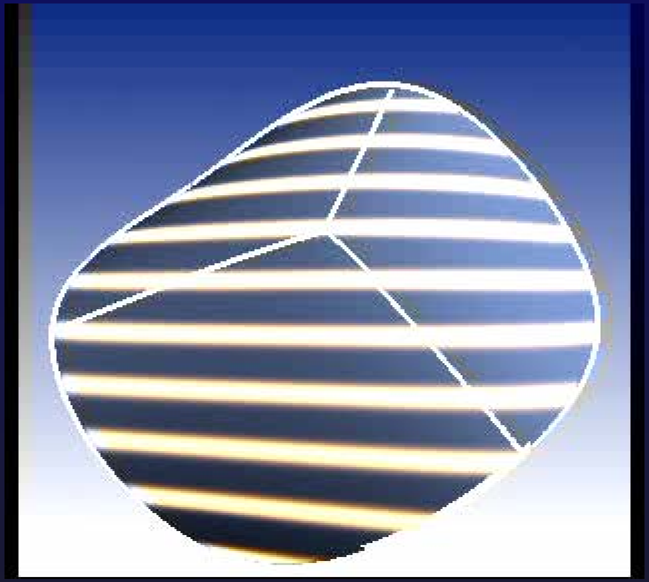
incorrect

(-1, - 0.5)

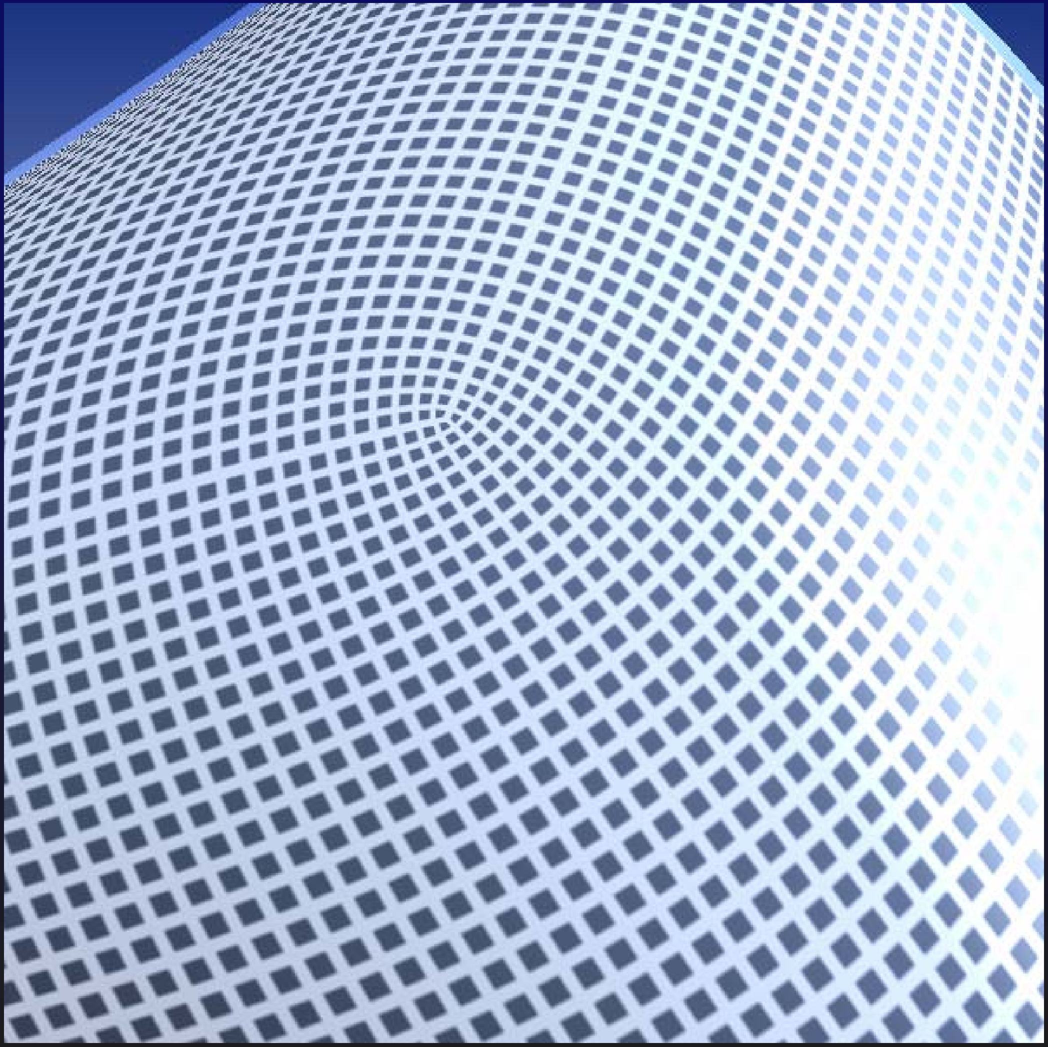
correct

(1,0.5)

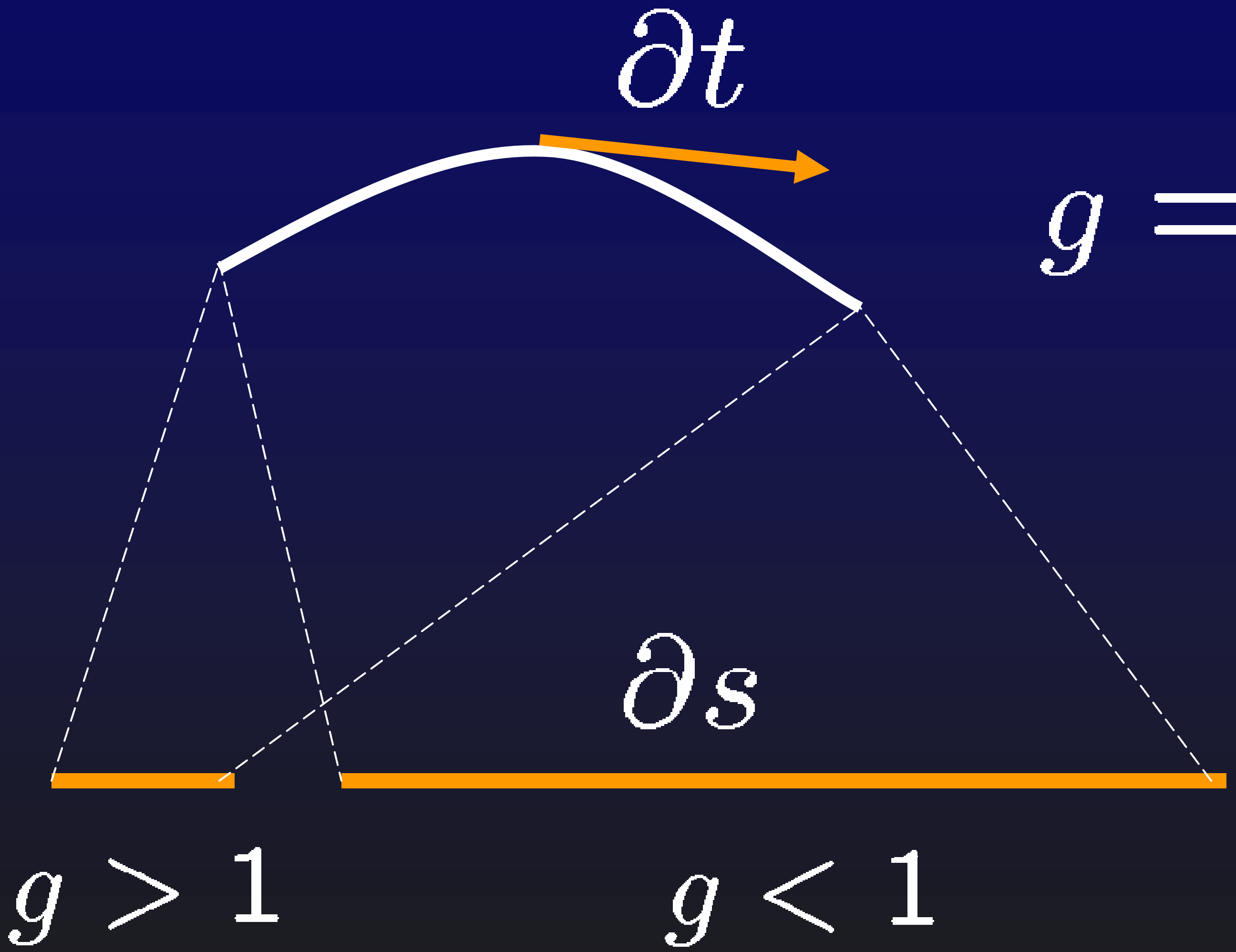
Are we Done ?



Distortions



Distortions (1D)



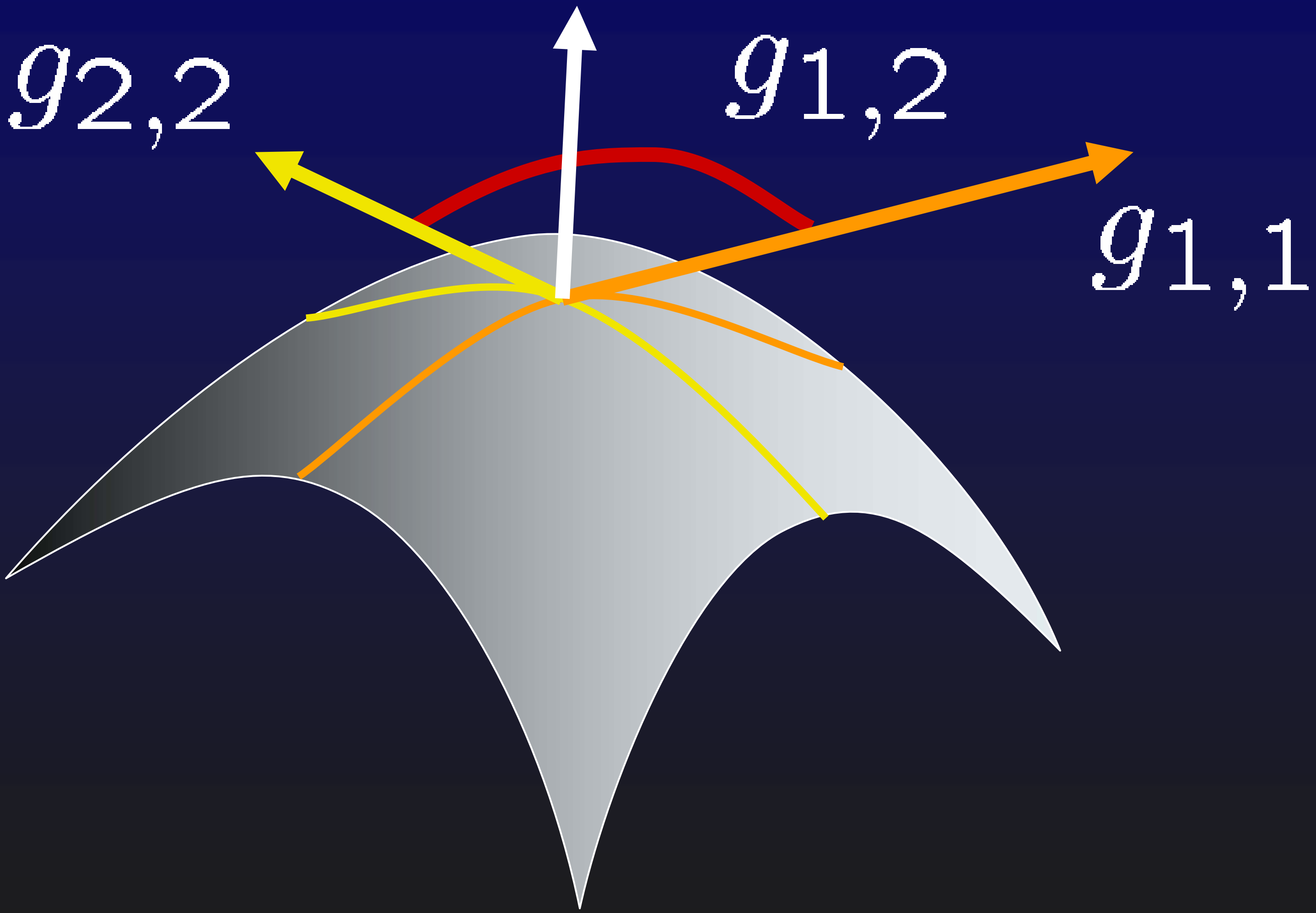
$$g = \frac{\partial t}{\partial s}$$

Distortions (1D)

$$\frac{\partial}{\partial t} = \frac{\partial s}{\partial t}\frac{\partial}{\partial s} = g^{-1}\frac{\partial}{\partial s}$$

$$\frac{\partial^2}{\partial t^2} = g^{-2}\frac{\partial^2}{\partial s^2}$$

Distortions (2D)



Distortions (2D)

Matrix instead of a scalar:

$$\mathrm{M} = \begin{pmatrix} g_{1,1} & g_{1,2} \\ g_{2,1} & g_{2,2} \end{pmatrix}$$

$$\mathrm{M}^{-1} = \begin{pmatrix} g^{1,1} & g^{1,2} \\ g^{2,1} & g^{2,2} \end{pmatrix}$$

Distortions (2D)

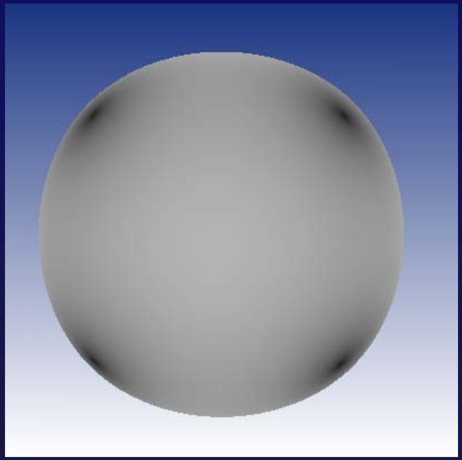
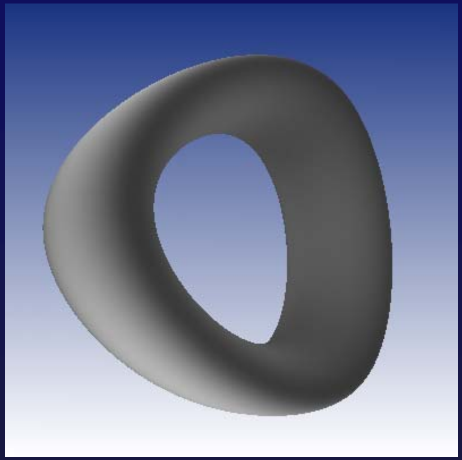
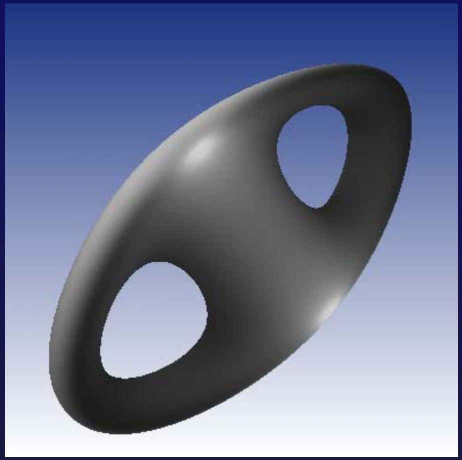
Operators in "Einstein Notation":

$$\nabla^i = g^{i,j} \frac{\partial}{\partial x^j}$$

$$\nabla^2 = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\sqrt{g} g^{i,j} \frac{\partial}{\partial x^j} \right)$$

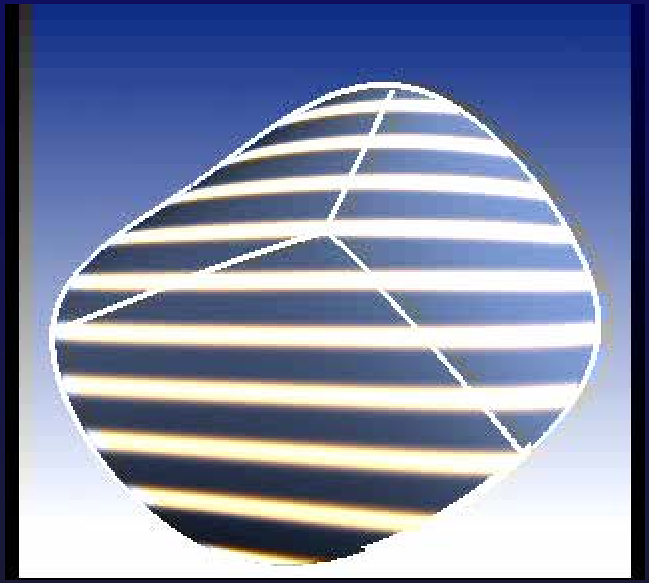
where $g = \det(M)$

Distortions (2D)

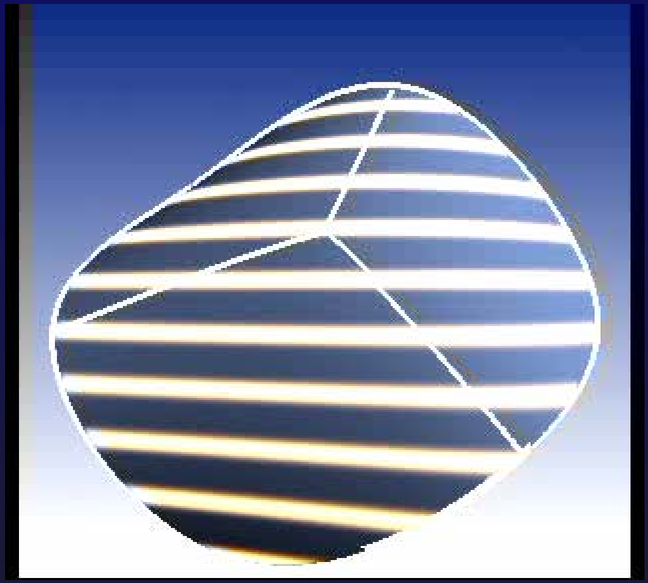





$$g = \det(M)$$

Distortions

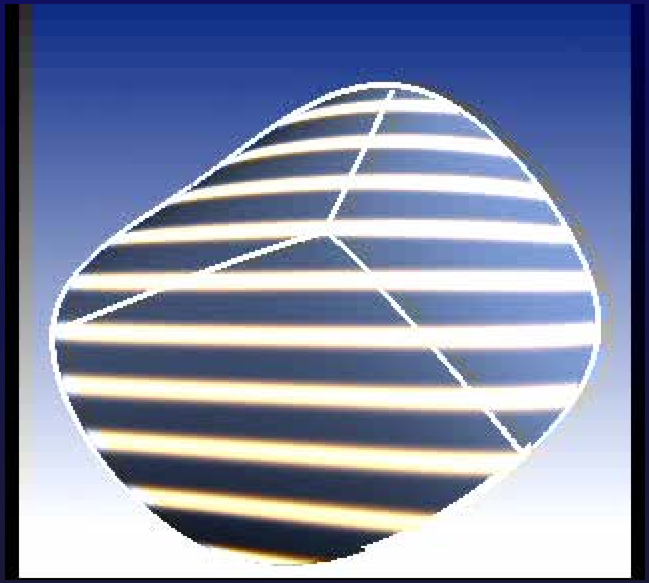


No metric

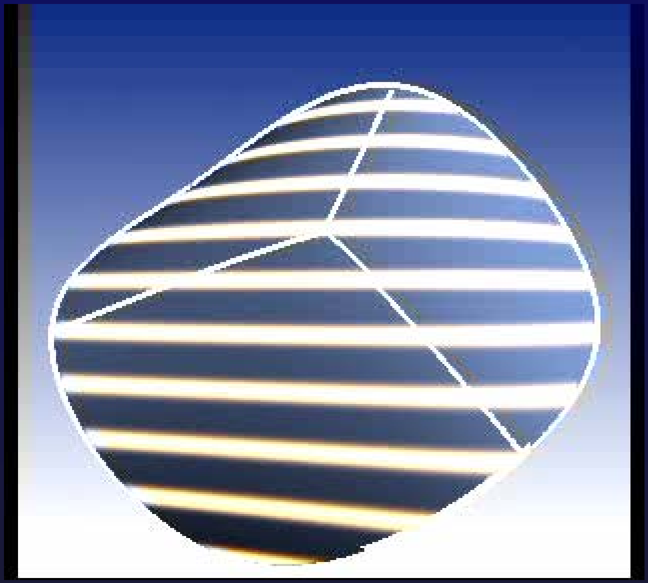


With metric

Distortions

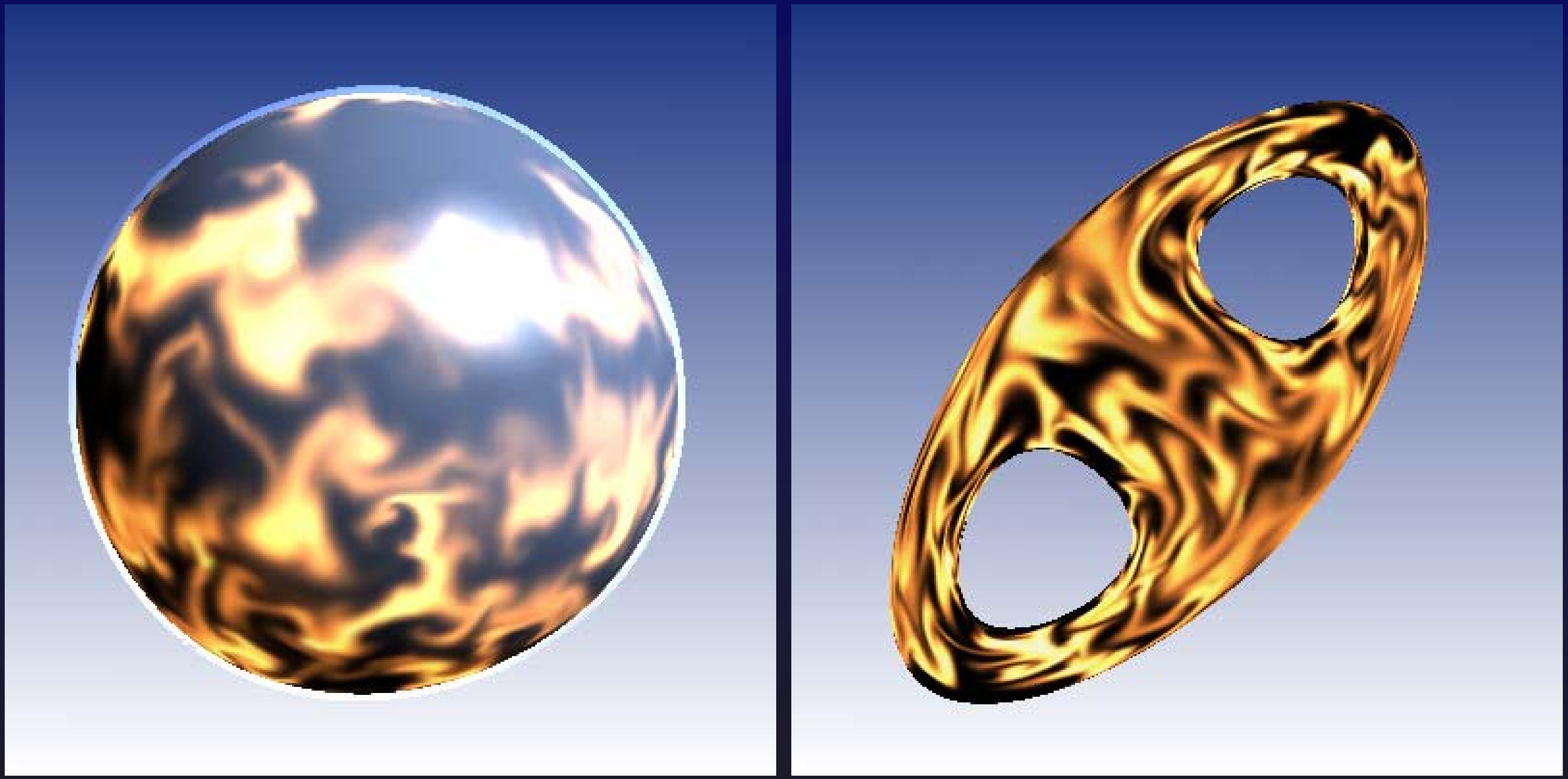


No metric

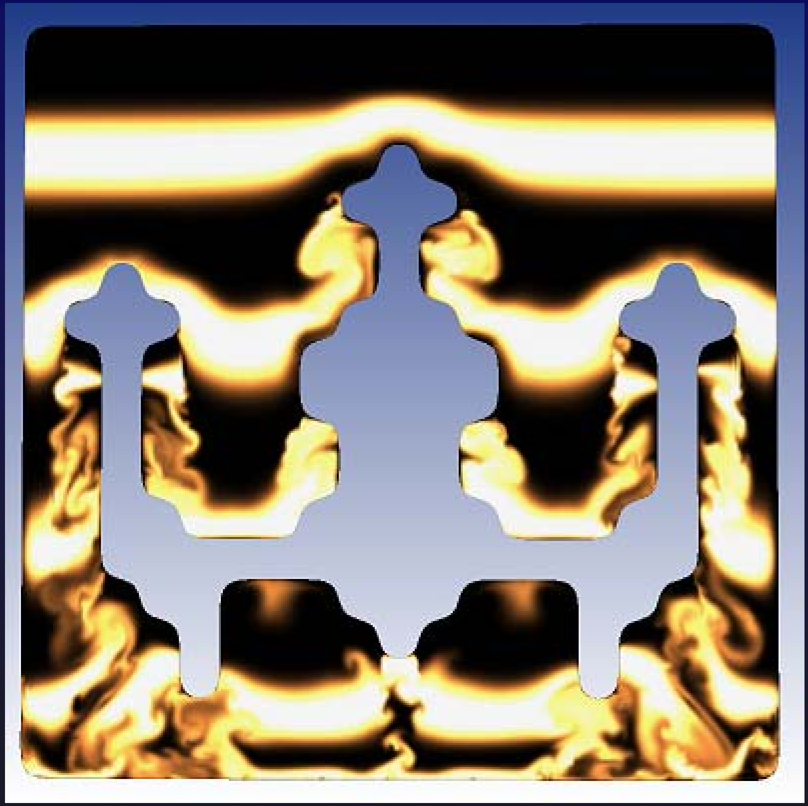


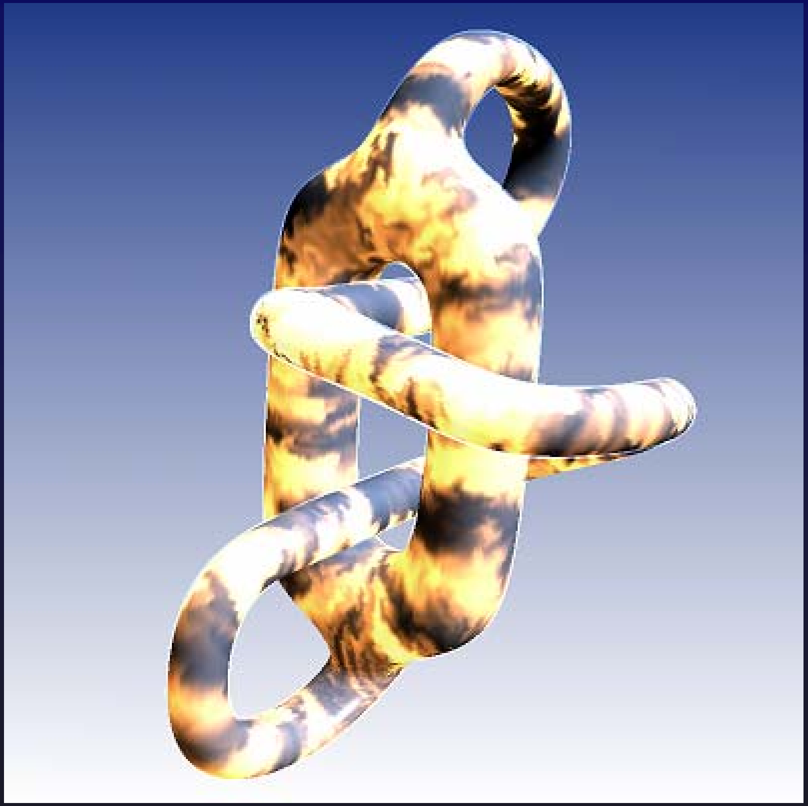
With metric

Results

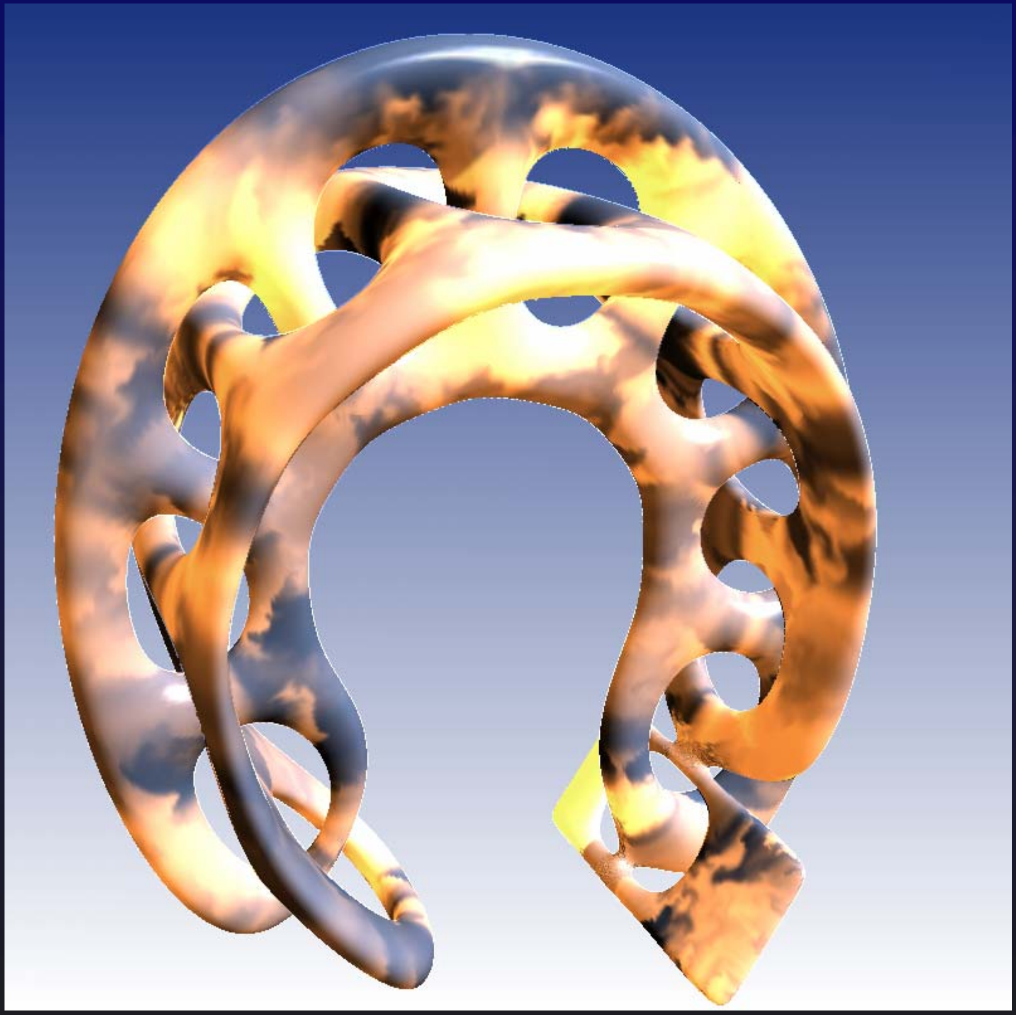


Results





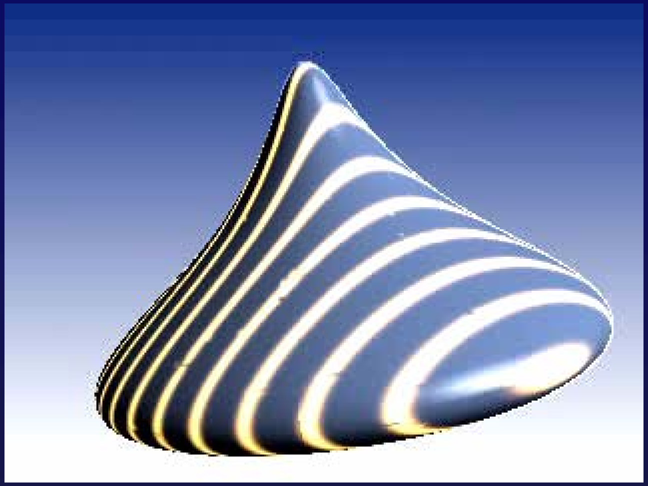
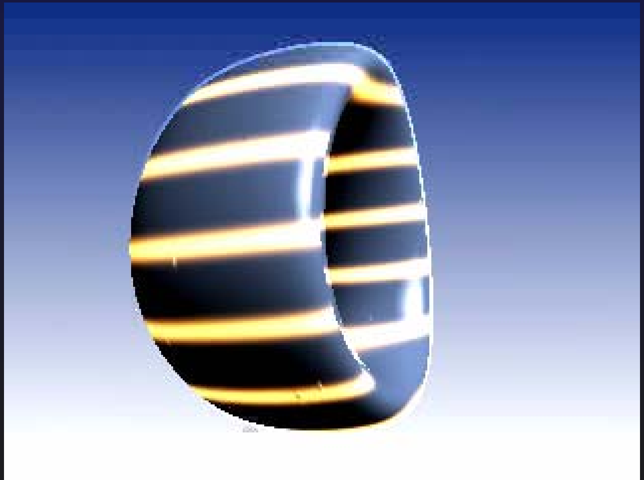
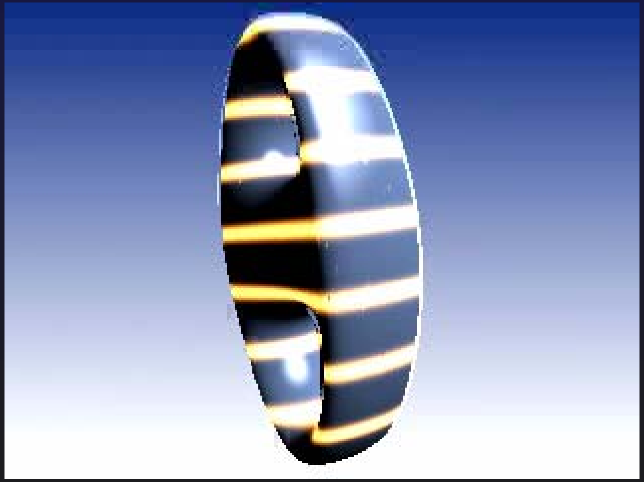
Results



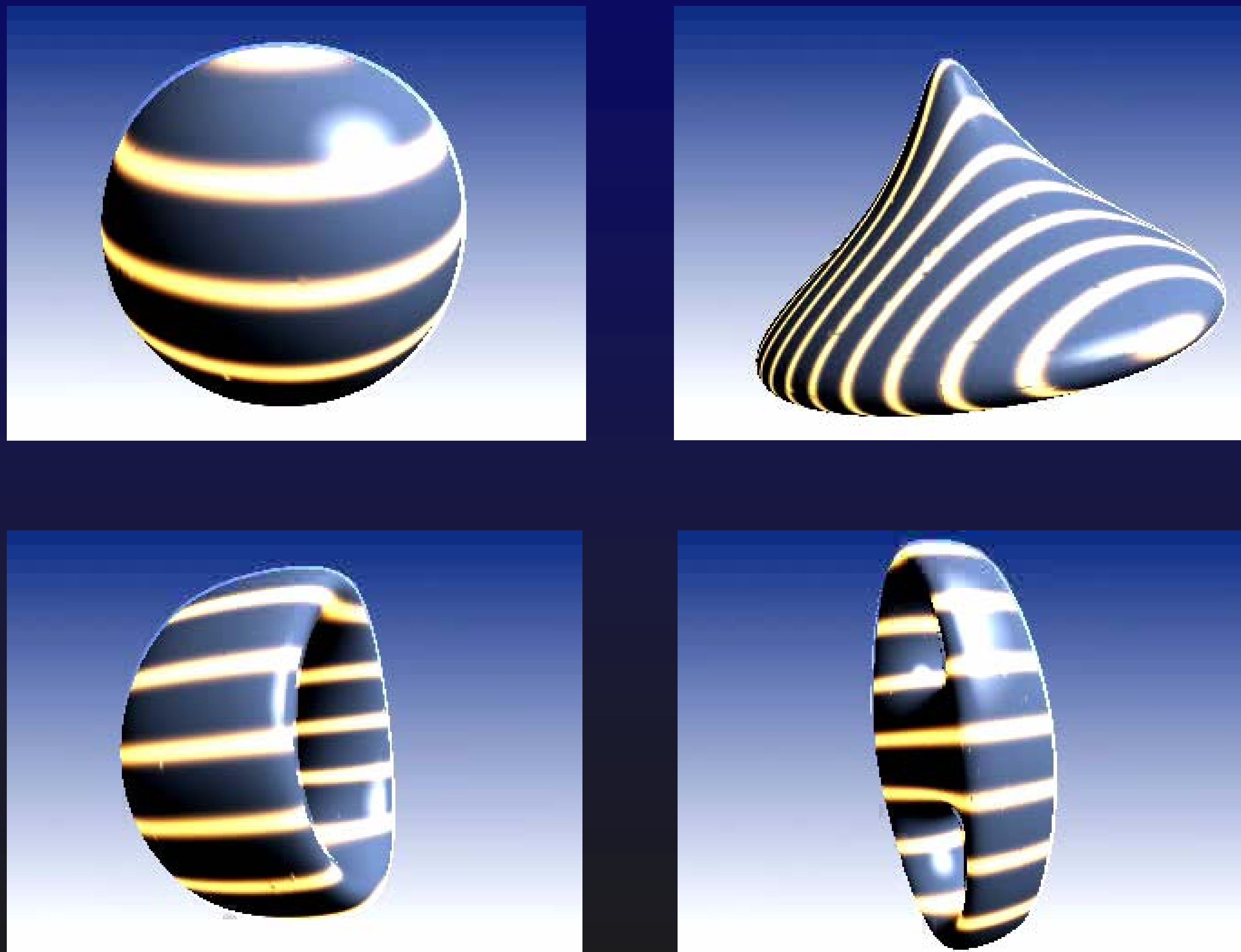
Results

Real-Time Demo

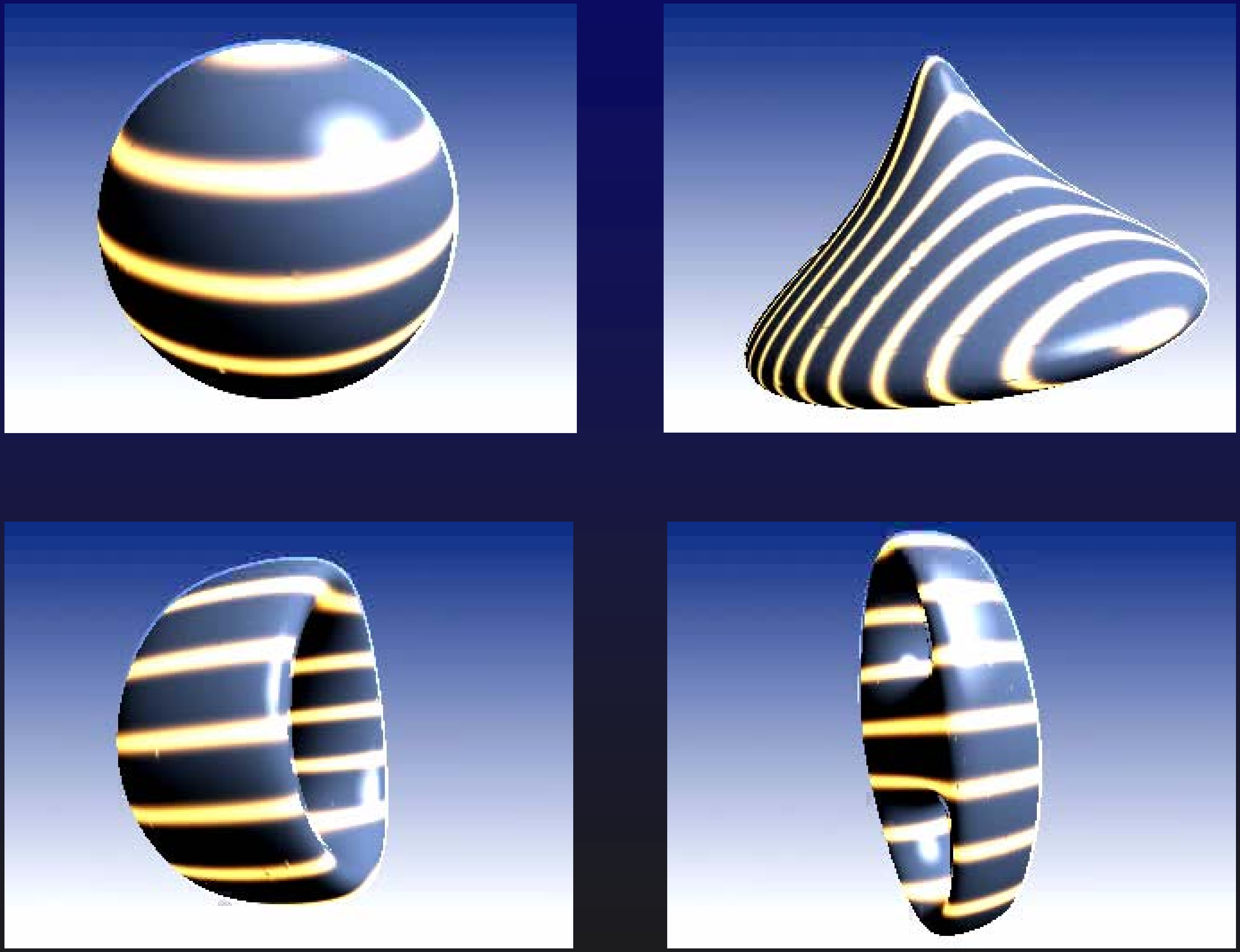
Results

Results



Results



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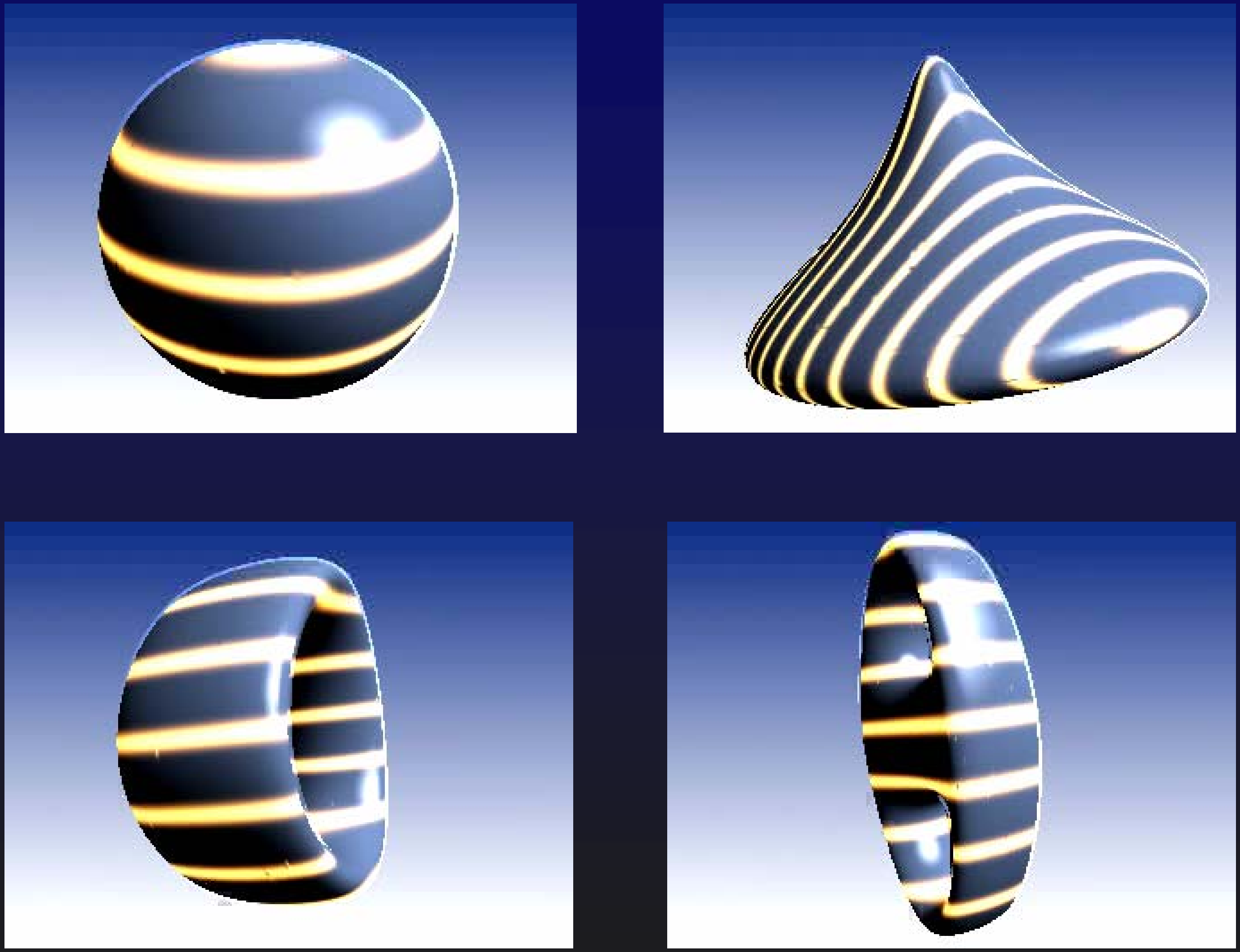

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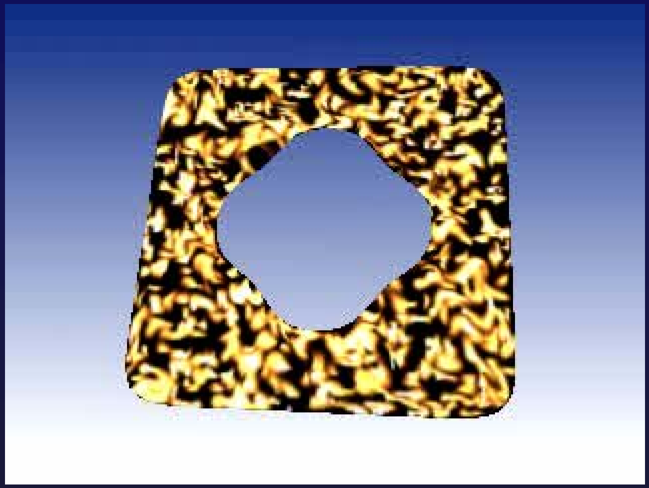

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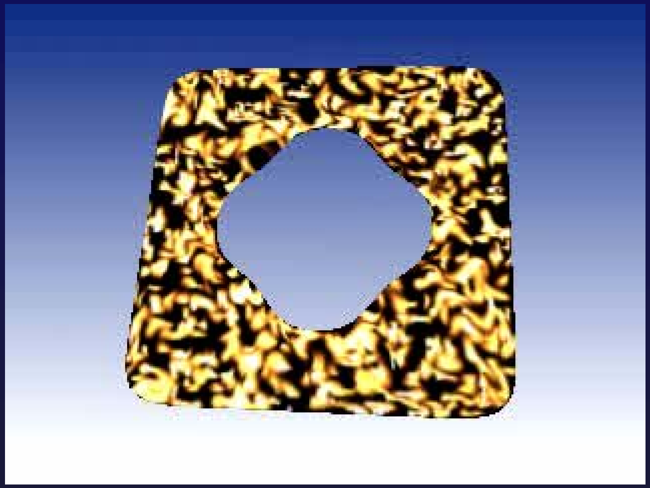
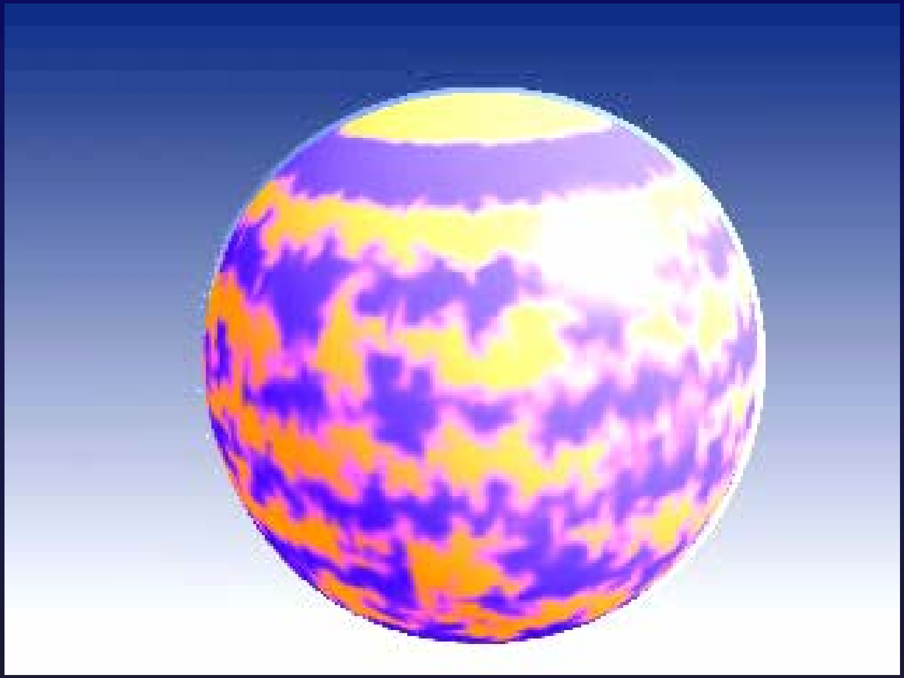


Results



+ Reaction-Diffusion

Other Surfaces

Loop Subdivision Surfaces

Bajaj (2003)

Implicit/Level Set

Bertalmio + Osher + Shapiro (2001-3)

Meshes

Desbrun, Alliez, Schroeder, CalTech, etc.

Future Work

Distortions still a problem

Other PDEs, processes, …

Extensions to 3D…