An Illumination Model for a Skin Layer Bounded by Rough Surfaces

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Alias | wavefront

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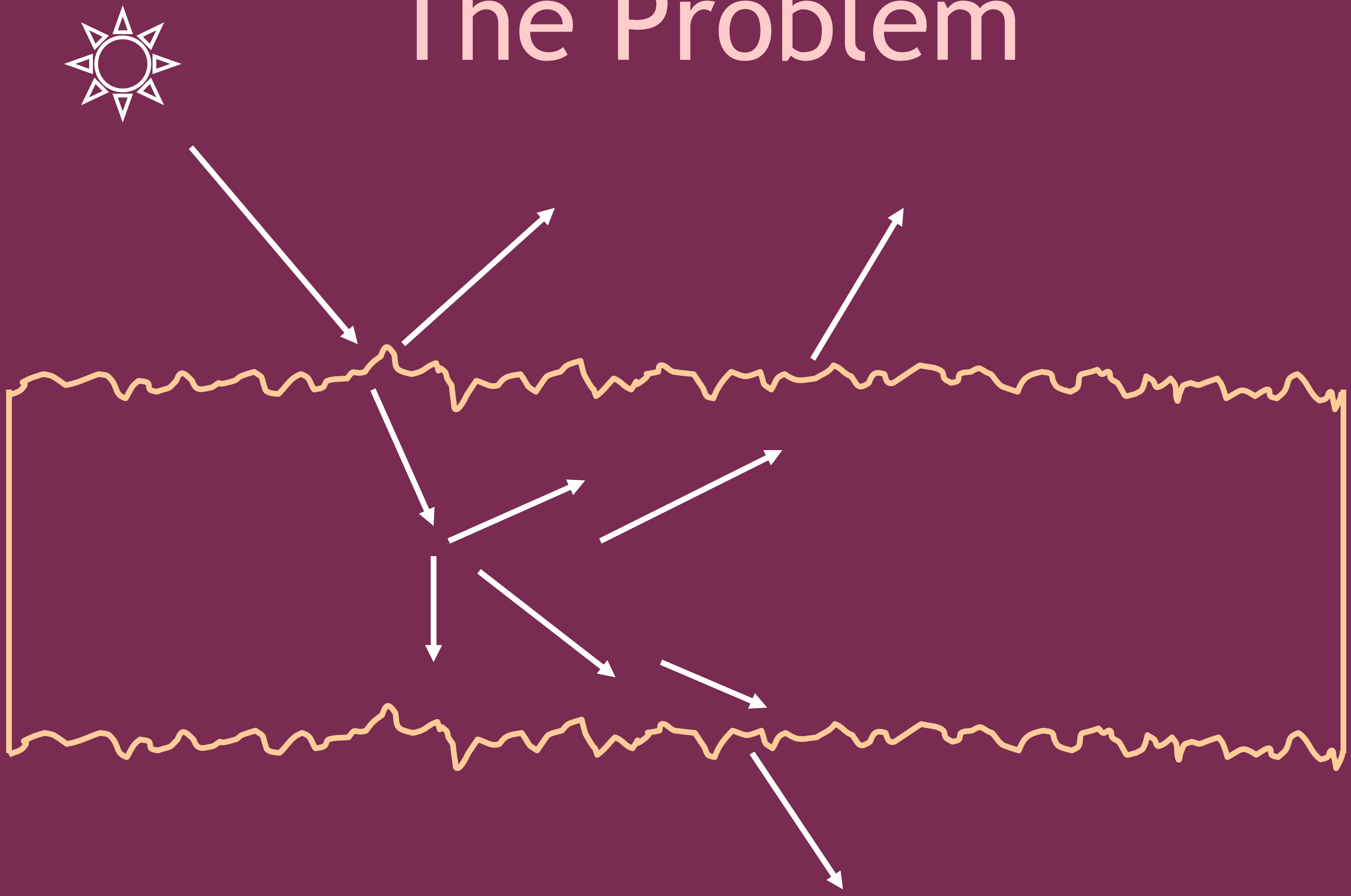
Motivation

Render effects of subsurface scattering (non-metals)

Two important examples:

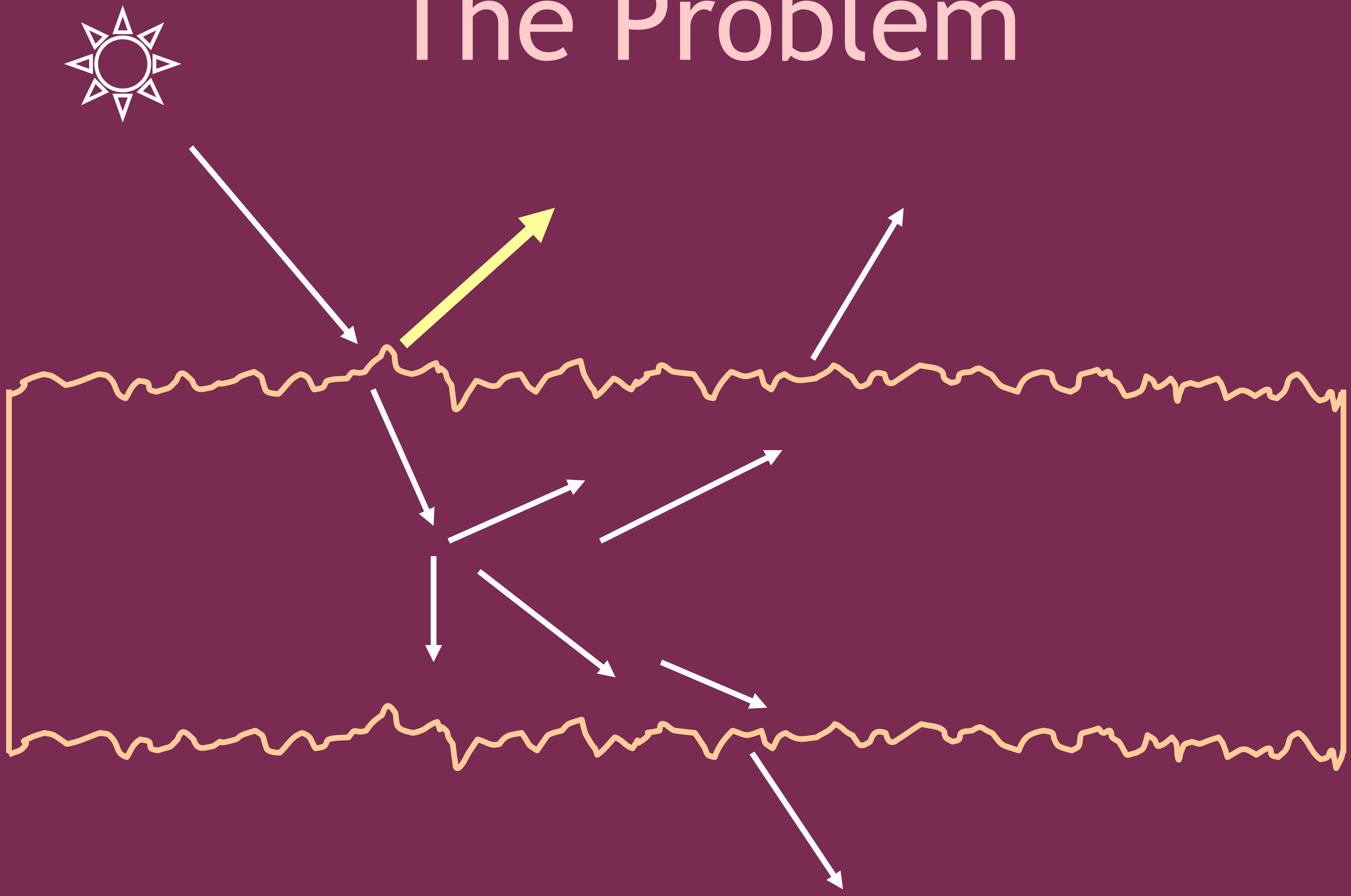
- Skin (Entertainment Industry)
- Paint (Automotive Industry)

The Problem



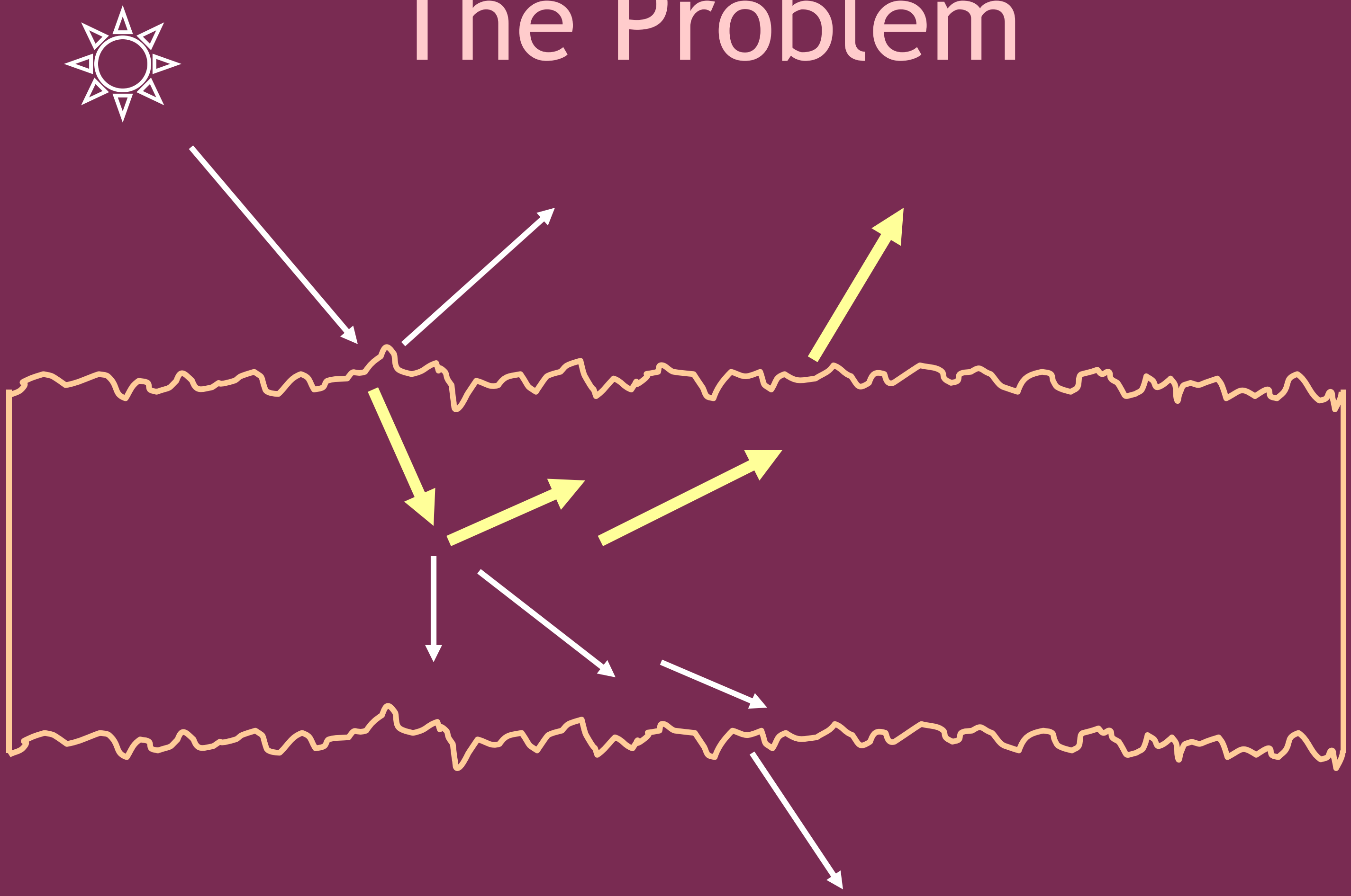
I = ...

The Problem



I = I_surface + ...

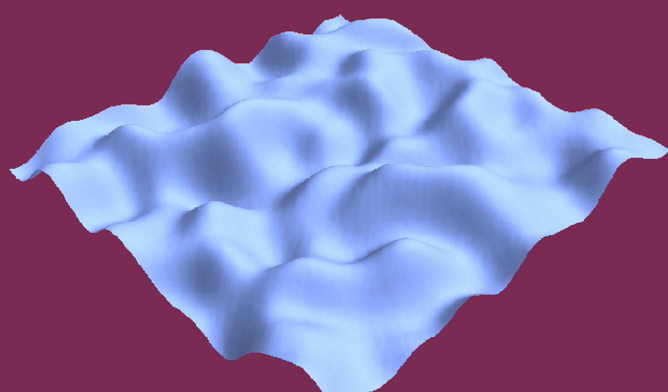
The Problem

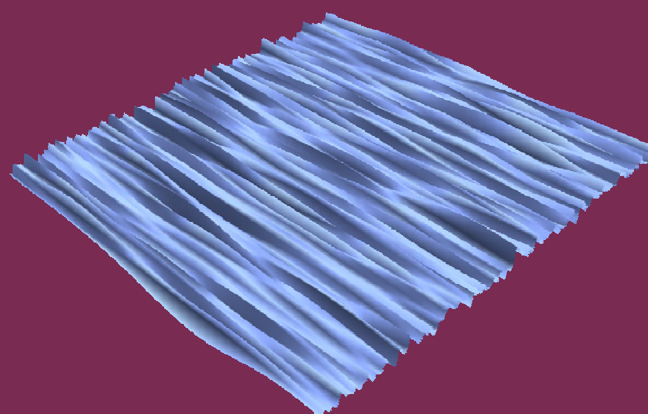


I = I_surface + I_subsurface

Surface Reflection & Refraction

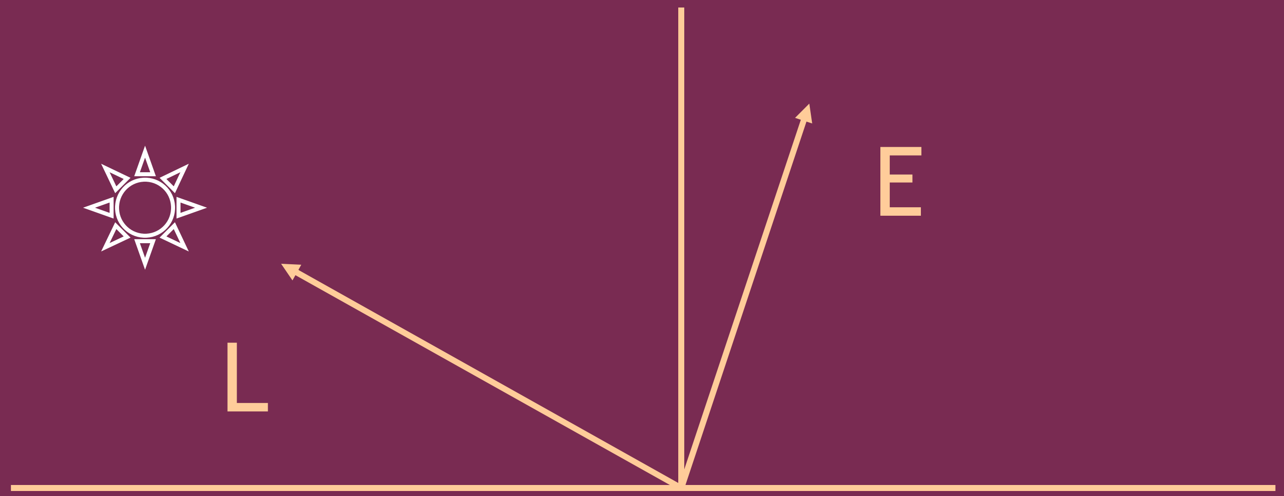
Use random surface height model





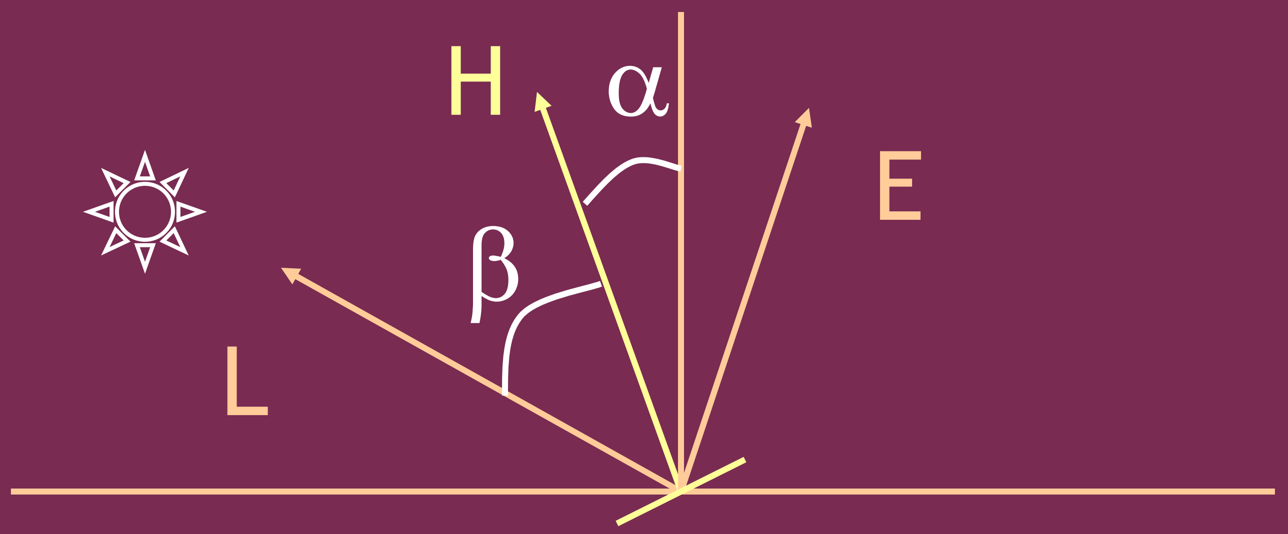
Cook-Torrance 81 (Beckmann)
He-Torrance 91 + Stam 99 (Wave Effects)

Reflection



Given L and E, find H

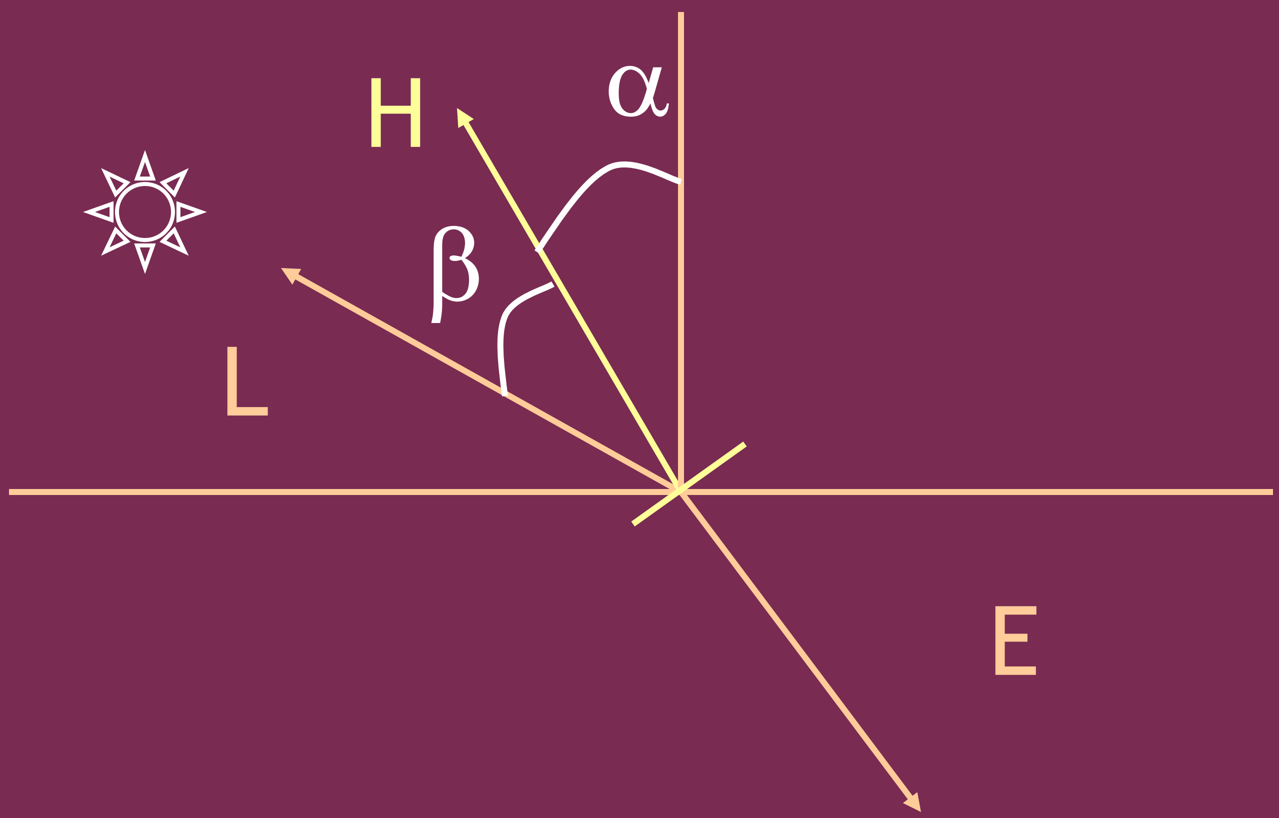
Reflection



Given L and E, find H

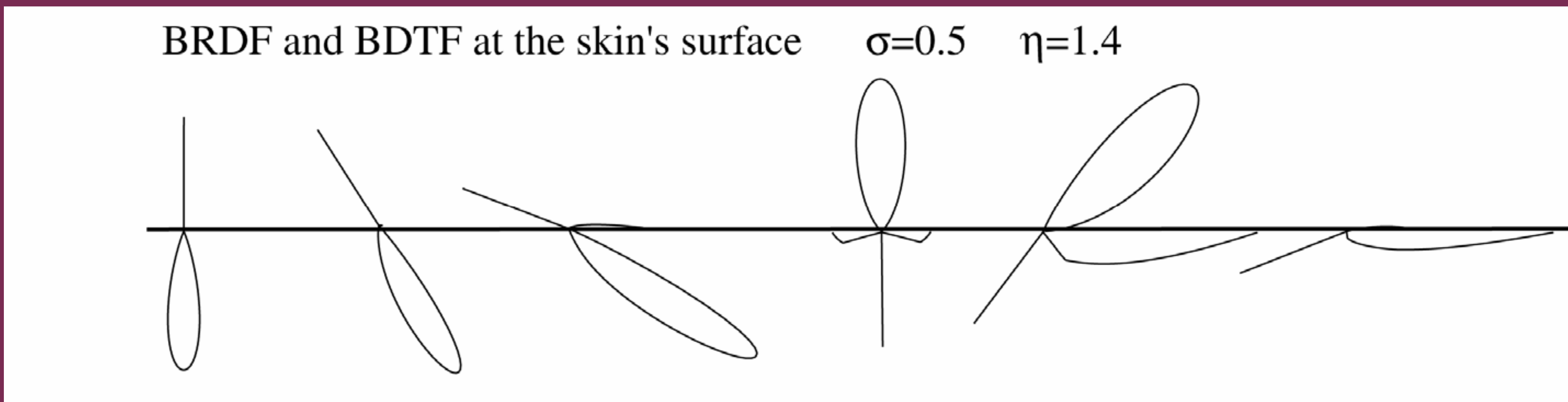
BRDF ~ Fresnel(β,η) Beck(α,σ)

Refraction

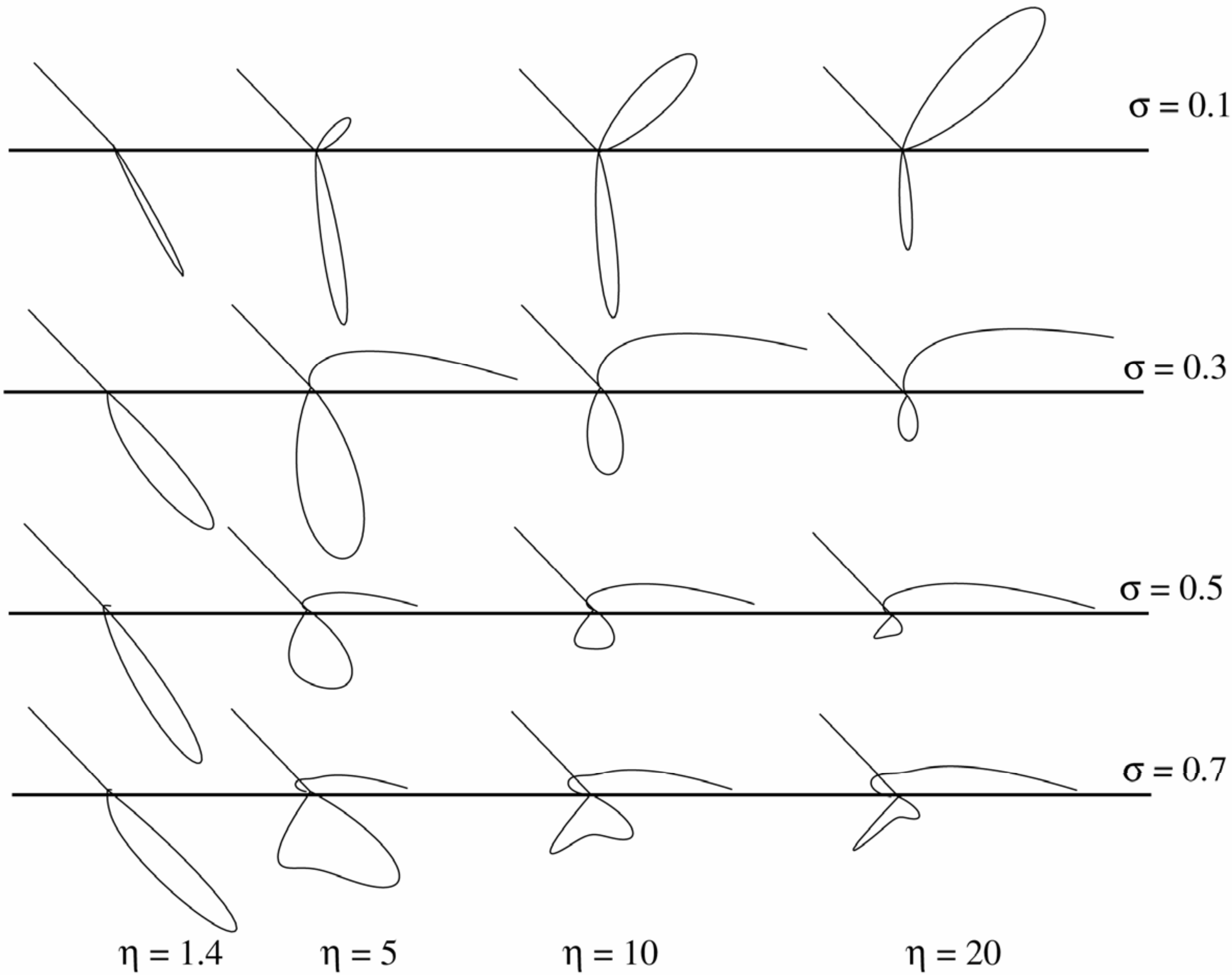


BTDF ~ [1-Fresnel(β,η)] Beck(α,σ)

Results



Varying the roughness σ and the ratio of indices of refraction η



Subsurface Reflection

Previous work:

Analytical: Lambert ??,
Hanrahan-Krueger 93
Monte-Carlo: Westin et al. 92,
Hanrahan-Krueger 93

BSSRDF: Pharr-Hanrahan 00,
Jensen et al. 01

Experiments

Monte Carlo too expensive

Analytical solutions unavailable

(show demo)

Our Solution

Semi-Analytical:

Solve discrete problem exactly
And fit splines to results

Setting

Homogeneous Layer bounded by rough surfaces

air

air / bone

Parameters of the Skin

surface
- Surface roughness : σ
- Index of refraction : $\eta \sim 1.4$

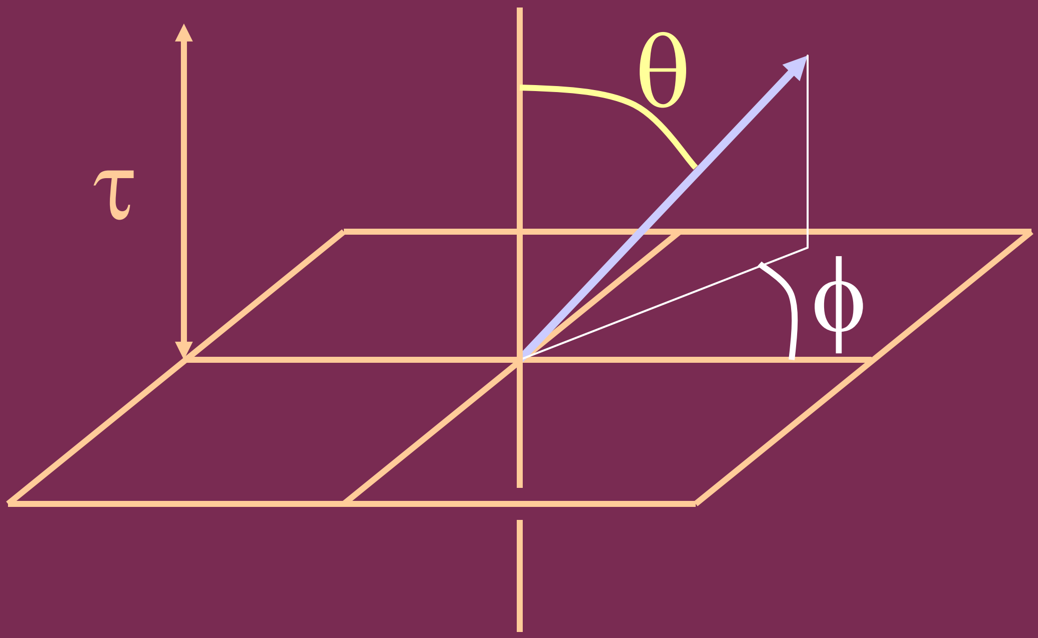
skin
- Transparency : $T = \exp(-\tau_b)$
  - τ_b : optical depth
- Albedo : Ω (amount of scattering)
- Anisotropy : g (forward scattering)

Human skin: $\Omega \sim 1$, $g \sim 1$, $T \sim 0$

Discretization

For a given incoming direction (θ_0,ϕ_0)

Intensity : $u(\tau,\theta,\phi)$



Discretization

For a given incoming direction (θ_0,ϕ_0)

Intensity : $u(\tau,\theta,\phi)$

Plan:

- Fourier Transform in ϕ
- Discretize angle θ
- Solve discrete problem

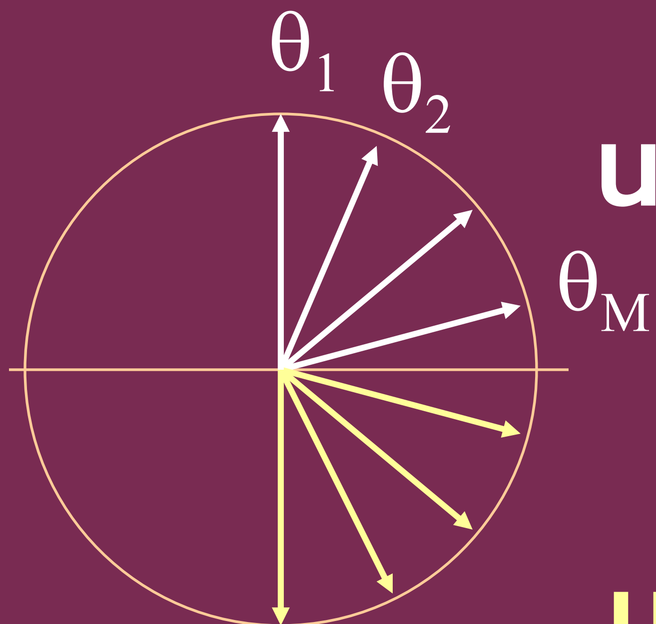
Fourier Transform in ϕ

$$u(\tau,\theta,\phi) = \sum_{k=0}^{N} u_k(\tau,\theta)\cos(2\pi k\phi)$$

(N+1) Independent Equations

-> only focus on one term

Discrete Ordinates for θ



$$\mathbf{u}^{+}(\tau) = \begin{pmatrix} u_1^{+}(\tau) \\ u_2^{+}(\tau) \\ \\ u_M^{+}(\tau) \end{pmatrix}$$

$\mathbf{u}^{-}(\tau)$

Equation of Transfer

$$\frac{d}{d\tau}\begin{pmatrix} u^+ \\ u^- \end{pmatrix} = -\begin{pmatrix} u^+ \\ u^- \end{pmatrix} + \begin{pmatrix} S^{++} & S^{+-} \\ S^{-+} & S^{--} \end{pmatrix}\begin{pmatrix} u^+ \\ u^- \end{pmatrix}$$

Change

out-scatter
+ absorption

in-scatter

Equation of Transfer

$$\frac{d}{d\tau}\begin{pmatrix} \mathbf{u}^{+} \\ \mathbf{u}^{-} \end{pmatrix} = \begin{pmatrix} \mathbf{M}^{++} & \mathbf{M}^{+-} \\ \mathbf{M}^{-+} & \mathbf{M}^{--} \end{pmatrix}\begin{pmatrix} \mathbf{u}^{+} \\ \mathbf{u}^{-} \end{pmatrix}$$

1D equivalent:

$$\dot{x} = a\,x \qquad x(t) = \exp(at)x_0$$

Diagonalize

$$\frac{d}{d\tau}\begin{pmatrix} w^+ \\ w^- \end{pmatrix} = \begin{pmatrix} L & 0 \\ 0 & -L \end{pmatrix}\begin{pmatrix} w^+ \\ w^- \end{pmatrix}$$

Solution:

$$w^+(\tau) = \exp(\ \tau L)\ w^+_0$$

$$w^-(\tau) = \exp(-\tau L)\ w^-_0$$

Diagonalize

$$\frac{d}{d\tau}\begin{pmatrix} \mathbf{w}^{+} \\ \mathbf{w}^{-} \end{pmatrix} = \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & -\mathbf{L} \end{pmatrix} \begin{pmatrix} \mathbf{w}^{+} \\ \mathbf{w}^{-} \end{pmatrix}$$

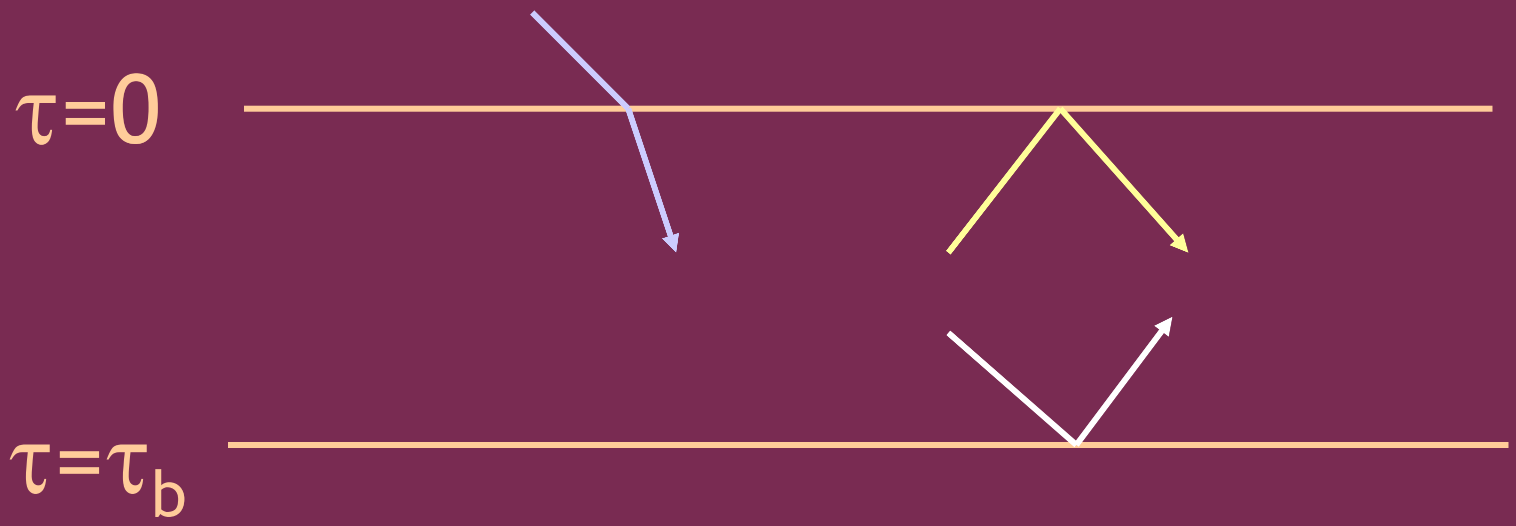
Solution:

$$\mathbf{w}^{+}(\tau) = \exp(\ \tau\mathbf{L})\ \mathbf{w}^{+}_{0}$$

$$\mathbf{w}^{-}(\tau) = \exp(-\tau\mathbf{L})\ \mathbf{w}^{-}_{0}$$

unknown

Find Unknowns



$$w^{-}(0) = \mathbf{t}_{12} + \mathbf{R}_{21}\, w^{+}(0)$$

$$w^{+}(\tau_b) = \mathbf{R}_{23}\, w^{-}(\tau_b)$$

Complete System

$$\begin{cases} \mathbf{w}^{+}(\tau) = \exp(\ \tau\mathbf{L})\ \mathbf{w}^{+}_{0} \\ \mathbf{w}^{-}(\tau) = \exp(-\tau\mathbf{L})\ \mathbf{w}^{-}_{0} \end{cases}$$

$$\begin{cases} \mathbf{w}^{-}(0) = \mathbf{t}_{12} + \mathbf{R}_{21}\ \mathbf{w}^{+}(0) \\ \mathbf{w}^{+}(\tau_b) = \mathbf{R}_{23}\ \mathbf{w}^{-}(\tau_b) \end{cases}$$

Linear System

$$\begin{cases} \mathbf{w}^{+}(\tau) = \exp(\ \tau \mathbf{L})\ \mathbf{w}^{+}_{0} \\ \mathbf{w}^{-}(\tau) = \exp(-\tau \mathbf{L})\ \mathbf{w}^{-}_{0} \end{cases}$$

$$\mathbf{w}^{-}_{0} = \mathbf{t}_{12} + \mathbf{R}_{21}\ \mathbf{w}^{+}_{0}$$

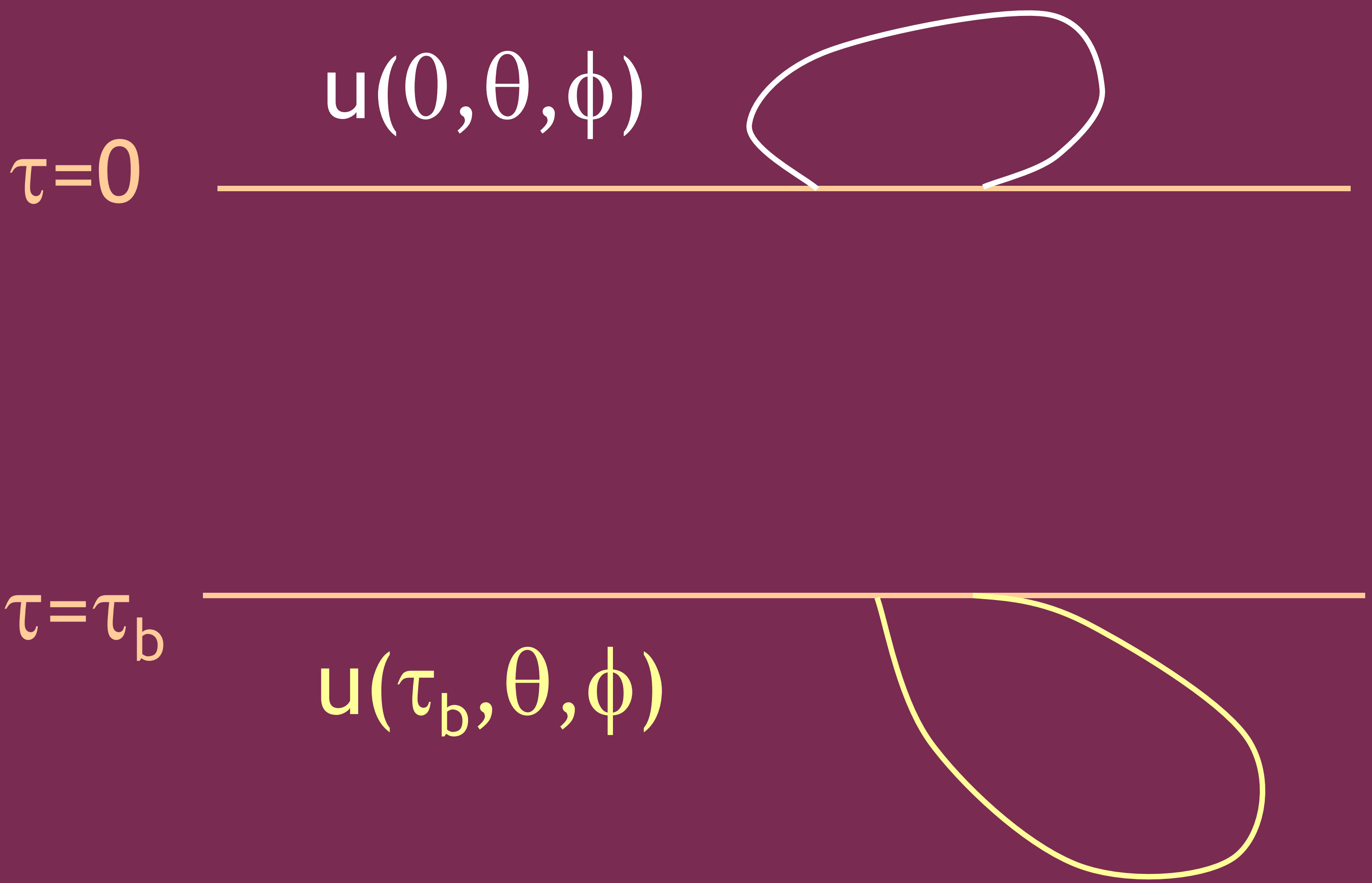
$$\exp(\tau_b \mathbf{L})\mathbf{w}^{+}_{0} = \exp(-\tau_b \mathbf{L})\ \mathbf{R}_{23}\ \mathbf{w}^{-}_{0}$$

Linear System

$$\begin{cases} \mathbf{w}^+(\tau) = \exp(\tau \mathbf{L})\, \mathbf{w}^+_0 \\ \mathbf{w}^-(\tau) = \exp(-\tau \mathbf{L})\, \mathbf{w}^-_0 \end{cases}$$

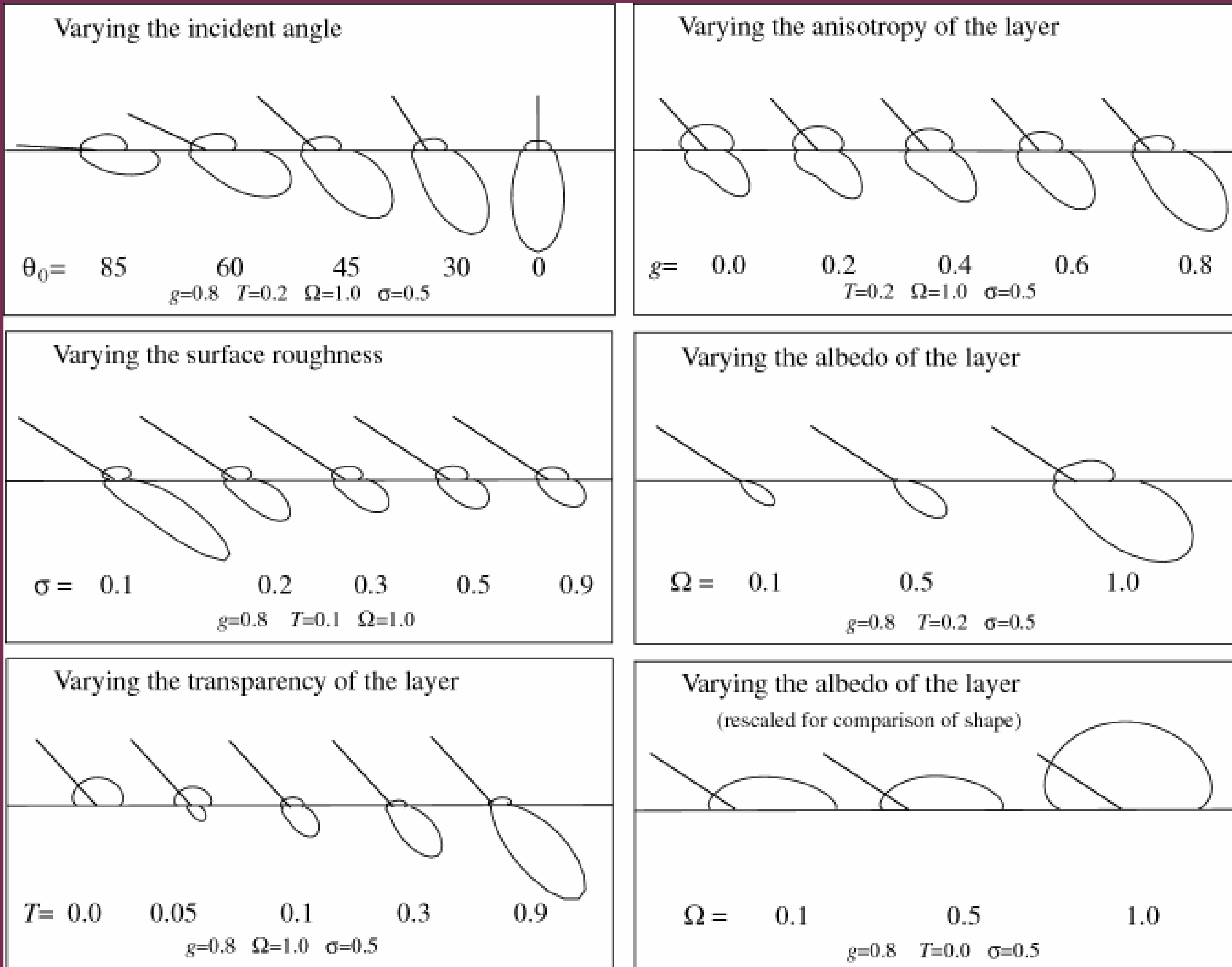
$$\begin{pmatrix} -\mathbf{R}_{21} & \mathbf{I} \\ \mathbf{I} & -\exp(-2\tau_b \mathbf{L})\, \mathbf{R}_{23} \end{pmatrix} \begin{pmatrix} \mathbf{w}^+_0 \\ \mathbf{w}^-_0 \end{pmatrix} = \begin{pmatrix} \mathbf{t}_{12} \\ \mathbf{0} \end{pmatrix}$$

Back to Reality



top	bottom	
$\mathbf{w}^{+}(0)$	$\mathbf{w}^{-}(\tau_b)$	
↓	↓	back from eigenspace
$\mathbf{u}^{+}(0)$	$\mathbf{u}^{-}(\tau_b)$	
↓	↓	fit splines
$u_k(0,\theta)$	$u_k(\tau_b,\theta)$	
↓	↓	add Fourier terms
$u(0,\theta,\phi)$	$u(\tau_b,\theta,\phi)$	

Results


Varying the incident angle
θ0= 85 60 45 30 0
g=0.8 T=0.2 Ω=1.0 σ=0.5
Varying the anisotropy of the layer
g= 0.0 0.2 0.4 0.6 0.8
T=0.2 Ω=1.0 σ=0.5
Varying the surface roughness
σ = 0.1 0.2 0.3 0.5 0.9
g=0.8 T=0.1 Ω=1.0
Varying the albedo of the layer
Ω = 0.1 0.5 1.0
g=0.8 T=0.2 σ=0.5
Varying the transparency of the layer
T= 0.0 0.05 0.1 0.3 0.9
g=0.8 Ω=1.0 σ=0.5
Varying the albedo of the layer
(rescaled for comparison of shape)
Ω = 0.1 0.5 1.0
g=0.8 T=0.0 σ=0.5

Results

Implemented as a MAYA shader

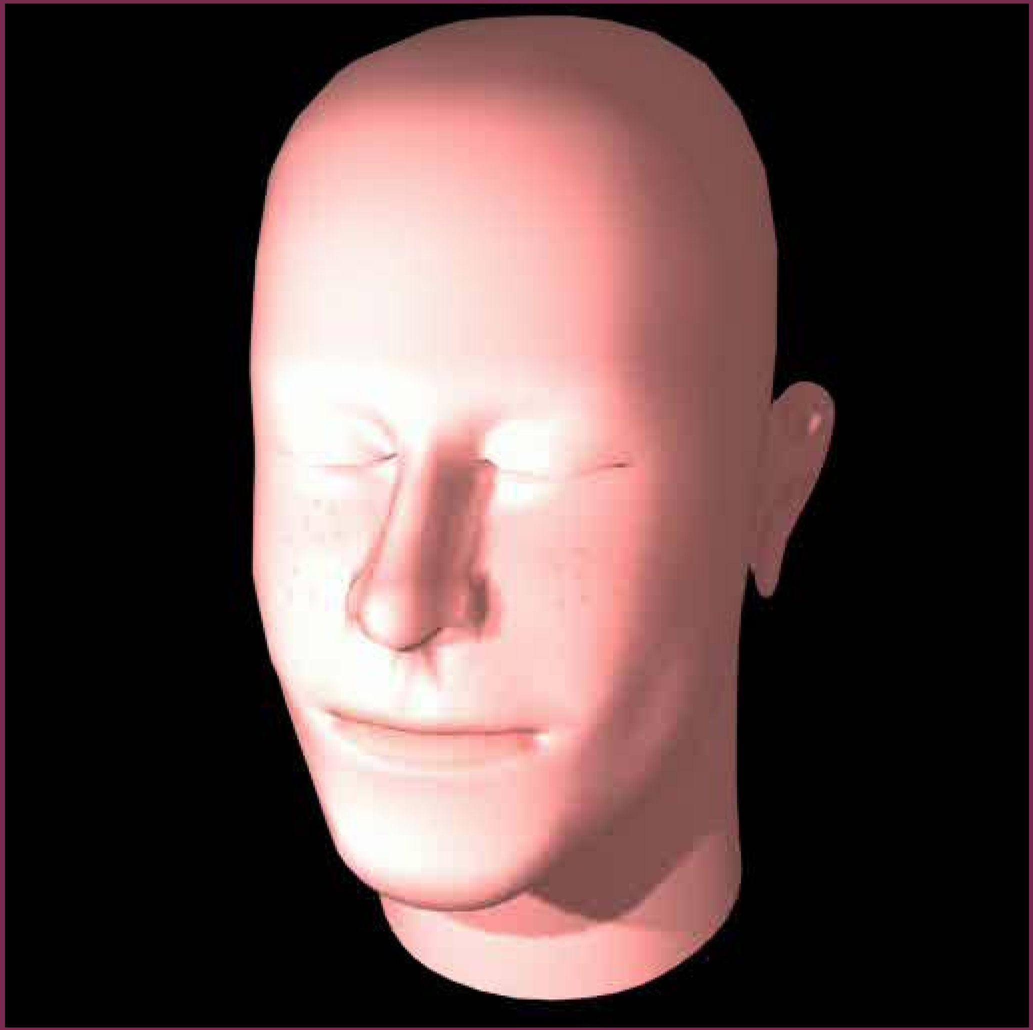
Available for free at

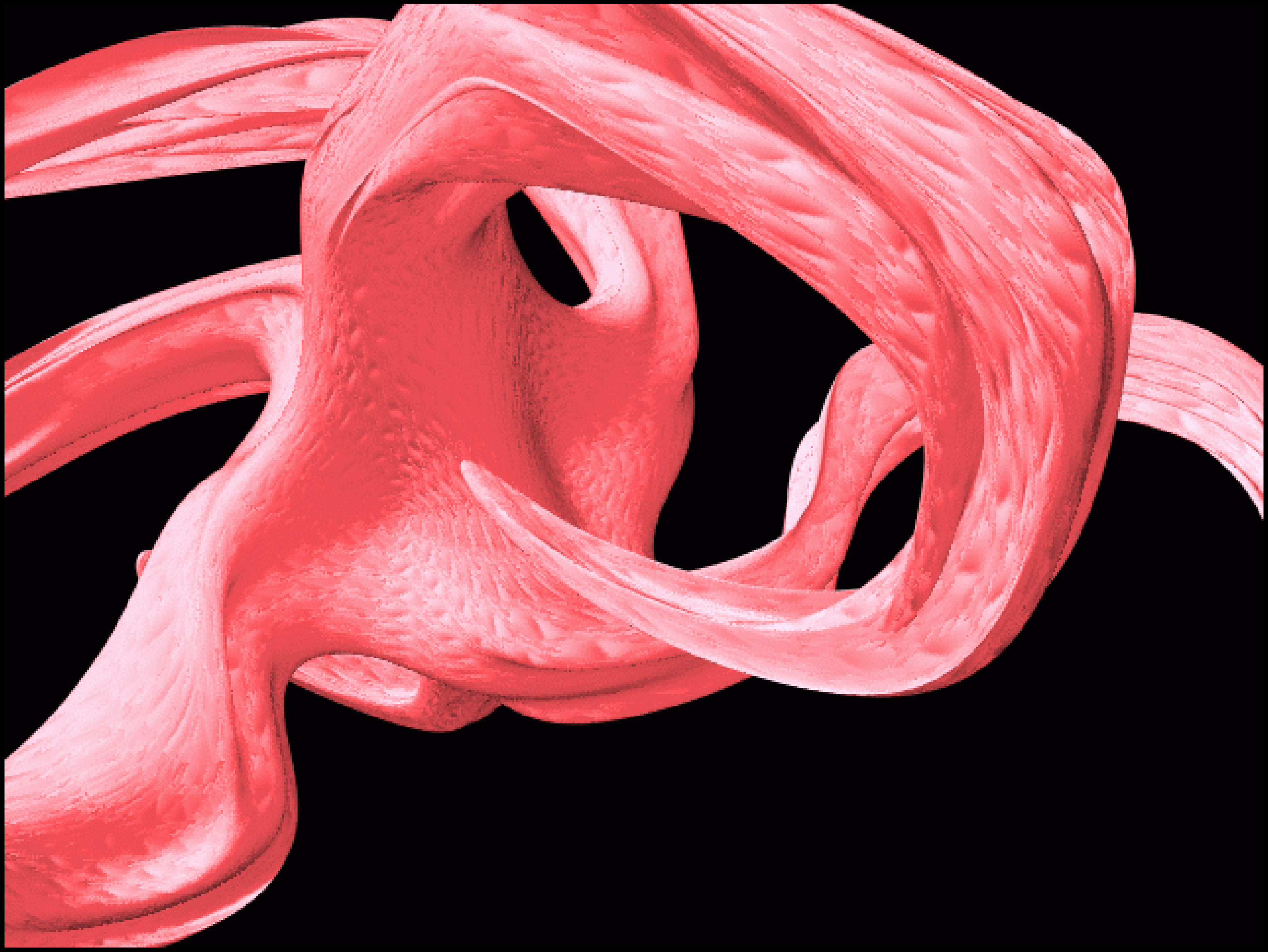
http://www.aliaswavefront.com

(follow “community” + “Download”)

Results

Lambert Hanrahan-Krueger New Model





Future Work

-Experimental validation
-More Layers
-Better fits
-Non-homogeneous layer
-BSSRDF